Homogenous and Heterogenous Contestants in Cardinal Tournament Games: Theory and Empirical Analysis

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Abstract

In this paper we show that sorting contestants in cardinal tournaments into more or less heterogenous groups creates different incentives for agents to exert effort. In particular we show that for a given mean of the tournament group's heterogeneity parameters, larger variance (more heterogenous agents) induces higher optimal effort. This implies that the principal can actually gain from heterogenizing the tournament groups. On the other hand, the effect of this change on growers' welfare is unclear because higher effort leads to higher productivity and hence higher payment, but also increases the cost of effort. Using broiler production contracts settlement data we empirically estimated a fully structural model of a cardinal tournament game with heterogenous players. Our counterfactual analysis shows that under reasonable assumptions the integrator's gain is actually larger than the growers' losses indicating that heterogenizing groups in cardinal tournaments may be efficient.

1 Introduction

Tournaments are labor contracts where an individual's payoff depends on own performance relative to others. There are *rank-order* or *ordinal tournaments* such as the ones considered by Lazear and Rosen (1981) and *cardinal tournaments* analyzed by Nalebuff and Stiglitz (1983), also known as the *yardstick competition* (Shleifer 1985) or the *piece-rate tournaments* (Tsoulouhas and Vukina 1999). The latter describe payment mechanisms where the reward is a continuous function (typically linear) of the difference between an individual player's performance and the group average performance.

Virtually all real world tournaments are contests among players with unequal abilities. When players have different abilities, rank-order tournaments are known to exhibit some undesirable properties. For example, asymmetries in the knowledge of abilities entail inefficiencies because contestants do not self-sort into their own homogeneous leagues. Correcting this may result in entry credentials and bigger prize spreads in leagues which target higher ability players. With full knowledge of abilities, rank-order tournaments with players of heterogeneous abilities still suffer from incentive problems. Handicapping and prize structures indexed by ability are possible consequences generated by a mixed tournament model in the presence of competition from segregated tournaments (see McLaughlin 1988).

It has been widely believed in the literature so far that, unlike the rank-order tournaments, cardinal tournaments exhibit no efficiency losses associated with mixing players of uneven abilities. When rewards are linearly related to performance, better players have no incentive to stop exerting effort once they realize that they are going to win and worse players have no incentive to surrender once they realize that they are going to lose. Consequently, payments by relative performance do not provide incentives for organizers to handicap better players or sort them into homogeneous groups. Under these tournaments, the incremental reward for improved performance (penalty for worse performance) at the margin is the same whether a player is more or less able (Knoeber and Thurman 1994).

The main tenet of this paper is that making groups of contestants in cardinal tournaments more homogenous or more heterogenous creates different incentives for agents to exert effort.

In general, somewhat counter-intuitively, we show that for a given mean of the tournament group's heterogeneity parameters, larger variance (more heterogenous agents) induces higher optimal effort. The aggregate efficiency (social surplus) of such a scheme depends on the particulars of the counterfactual analysis, but the principal always wins by mixing contestants of different abilities rather then sorting them into more homogenous groups.

The fact that mixing players of unequal abilities in tournament setting may have certain welfare ramifications was also discovered by Levy and Vukina (2004). They focused on the agents' welfare and defined the league (group) composition effect as the change in the distribution of tournament payoffs which results from an exogenous assignment of players to heterogeneous groups in which they compete. Using the data on broiler production contract settlements, they demonstrated that since the estimated variance of common shocks exceeded the variance of the growers idiosyncratic shocks, the payments to growers in a tournament had less variance than under a simple piece rate for a one-time tournament and also for multiple tournaments when leagues are randomly composed. However, when payments are made over a time horizon with fixed leagues, a simple piece rate contract offered less variance than any tournament given a long enough time horizon.

In addition to its theoretical contribution, this paper also contributes to the growing literature on structural econometrics approach to estimating tournament models, which proves to be quite useful for conducting welfare simulations. Recently, several papers have estimated structural models of various type tournaments. Shum (2007) estimated an elimination tournament model to explain intra-firm wage differentials in the retail industry. Zheng and Vukina (2007) estimated a rank-order tournament model to quantify the efficiency gains of an organizational innovation that would replace an ordinal tournament with a cardinal one. Finally, Vukina and Zheng (2007) provided a methodology for estimating rank order tournament models with private information. This paper represents the first attempt to structurally estimate a cardinal tournament model that captures the most important features of the production contracts observed in the broiler chickens industry. Using broiler production contracts settlement data of similar structure to the data set used by Levy and

Vukina (2004), we empirically quantified the welfare effects of heterogenizing tournament groups. The estimation results and the subsequent counterfactual analysis show empirically plausible results.

The rest of the paper is organized as follows. In the next section we describe the essential features of broiler production contracts and introduce the data set. In Section 3 we introduce our model of a cardinal tournament and derive the main theoretical result. Section 4 is devoted to the structural estimation methodology and the presentation of results. In Section 5, we quantify the welfare effects of heterogenizing the tournament groups and Section 6 concludes.

2 Industry and Data

The broiler industry is often considered a role model for the industrialization of agriculture. The industry is entirely vertically integrated from breeding flocks and hatcheries to feed mills, transportation divisions and processing plants. The final (finishing) stage of production where one day old chicks are brought to the farm and then grown to market weight is organized almost entirely through contracts between integrators and independent growers. Large national companies, such as Tyson Foods, Pilgrim's Pride, or Perdue Farms dominate broiler contract production. These companies run their operations through smaller divisions spread throughout the country, but mainly in the south-east.

Modern broiler production contracts are agreements between an integrator company and growers that bind farmers to tend for company's chickens until they reach market weight by strictly following specific production practices in exchange for monetary compensation. According to a typical contract, the grower provides land, housing facilities, utilities (electricity and water) and labor and pays for operating expenses such as repairs and maintenance, clean-up, and manure and mortality disposal. The company provides chicks, feed, medication, and the services of field men. Most of the modern broiler contracts are settled using a two-part piece-rate tournament consisting of a fixed base payment per pound of live meat produced and a variable bonus payment based on the grower's relative performance. The bonus payment is determined as a percentage of the difference between group average settlement costs and producer's individual settlement costs. Settlement costs are obtained by adding chicks, feed, medication, and other customary flock costs divided by total pounds of live poultry produced. The calculation of the group average performance includes growers whose flocks are settled on the same date. The time between two settlement dates typically does not exceed two weeks. For the below average settlement costs (above average performance), the grower receives a bonus and for the above average settlement costs, he receives a penalty (for details see for example Levy and Vukina, 2004).

The total payment R_i to grower i is the sum of the base payment and the bonus factor multiplied by the live pounds of poultry moved from the grower's farm:

$$R_i = \left[a + b \left(\frac{1}{n} \sum_{j=1}^{n} \frac{c_j}{y_j} - \frac{c_i}{y_i} \right) \right] y_i \tag{1}$$

where y_i is the quantity produced, a is the base payment (e.g., 3.5 - 4.5 cents a pound), $\frac{c_i}{y_i}$ is an individual grower's settlement cost, and b is the marginal bonus payment expressed as a percentage of the production cost savings that the grower retains.

Different companies, or different profit centers within the same company, typically specialize in the production of a particular size (weight) birds and offer their own contracts to their growers. The contracts for growing different size birds usually differ only with respect to the base payment (parameter a in (1)) in that farmers growing heavier birds typically receive larger base payment than those growing smaller birds. An interesting feature of the broiler contracts is that they are explicitly uniform and short-term (one flock of birds at a time). All growers, growing the same size birds for the same profit center, receive an identical contract regardless of their past performance, the length of tenure with the company, or any other specific attribute. The composition of the tournaments (settlement groups) is governed by timing and logistics of the production process and not with an attempt to form more homogenous or more diverse groups of contestants.

The data set used in this study includes broiler production information gathered from the payroll data of one company's profit center whose production contract corresponds to the payment scheme described in (1). Each observation in the data set represent one contract settlement, i.e., the payment received and the grower performance associated with one grower and one flock of birds delivered to the integrator's processing plant. The data comes from the so called settlement sheets and contain the information on the quantities and costs of various inputs supplied by the integrator (chicks, feed, medication, vaccination etc.), the number of birds placed and harvested, the quantity of broiler meat (live weight) produced, the dates when production started and terminated, mortality rates, etc.

The tournaments are separated by the settlement date, which happened to be once a week. For our contract, the settlement dates range from July 1995 to July 1997 totalling 104 tournaments each. The total number of growers is 356, and the number of observations (settlements) per grower varies from one to twelve flocks. The total number of usable observations is 3,247 flocks. The average live weight of the fully grown broilers is 4.81 pounds with a maximum of 5.75 pounds and a minimum of 3.88 pounds, the average number of days that a grower needs to grow chickens to that weight is 53 with a maximum of 79 and a minimum of 43 and the average feed conversion ratio is 2.03 with a maximum of 3.38 and a minimum of 1.83.

3 The Cardinal Tournament Model

Consider a $N-$player tournament game in which N risk-neutral growers contract with a risk-neutral integrator the production of broiler chickens. Each grower i $(i = 1, 2, ..., N)$ is given the same combination of inputs (chicks and feed) denoted by D and normalized to be \$0.01, or 1 cent.[1] Given D, the output of grower i is specified as

$$y_i = \theta_i e_i u_i \eta \tag{2}$$

[1] Here we assume constant returns to scale production technology and therefore this normalization is innocuous. We also assume that the combination of chicks and feed is feasible, i.e., it reflects the target weight of finished broilers and nutritionally meaningful feed-conversion ratio.

where y_i is the pounds of live chicken weight, e_i is grower i's effort and θ_i is her idiosyncratic ability (efficiency) parameter. We define the grower ability in a broad sense as inherent or acquired skills resulting from experience, education, age, etc., as well as other grower-specific factors such as location, quality, and vintage of the production facilities and equipment.[2] Higher θ_i implies that a grower can combine inputs and effort more efficiently in the production of broiler meat. We assume that from grower i's perspective, θ_j, $\forall j \neq i$, the abilities of other growers in the same tournament, are random variables drawn from a distribution $G(\cdot)$ with support $[\underline{\theta}, \overline{\theta}]$ and that $\underline{\theta} \geq 0$. Distribution $G(\cdot)$ is twice continuously differentiable and has density $g(\cdot)$ that is strictly positive on the support. This specification captures the real-life situation where growers typically do not know who their opponents in a particular tournament are, but know the distribution of other growers' abilities through repeated participation in similar tournaments for an extended period of time.

The stochastic production technology is characterized by two types of shocks. Both grower i's idiosyncratic productivity shock u_i (equipment failure, sick child, etc.) and the common productivity shock η (outside temperature, humidity, feed formula, etc.) materialize slowly during the production process. Shocks u_i and η are assumed to be drawn from distributions $F(\cdot)$ with support $[\underline{u}, \overline{u}]$ and $\underline{u} \geq 0$ and $P(\cdot)$ with support $[\underline{\eta}, \overline{\eta}]$ and $\underline{\eta} \geq 0$, respectively. Both $F(\cdot)$ and $P(\cdot)$ are twice continuously differentiable and have densities $f(\cdot)$ and $p(\cdot)$ that are strictly positive on the support. Each grower only learns u_i and η after the production process is complete but it is common knowledge that the two shocks are drawn from the two densities. Finally, we assume that θ_i, u_i and η are independent of each other.

The grower performance is determined by measuring how much output (pounds of live chicken weight) can she produce with \$1 worth of inputs,

$$f_i = \frac{D}{y_i} = \frac{1}{\theta_i e_i u_i \eta}. \tag{3}$$

[2]For example, one grower may outperform her peers because she is younger and better educated but also because her chicken houses are equipped with tunnel ventilation. Tunnel ventilation is known to work better than the standard curtain ventilation in summer months when the climate is hot and humid.

Combining (1) and (3) the grower payment can be written as

$$R_i = \left[a + b\left(\frac{1}{N}\sum_j f_j - f_i\right)\right] y_i. \tag{4}$$

Growers are assumed to be risk-neutral, hence grower i's payoff function is given by

$$\pi_i = R_i - C(e_i) \tag{5}$$

where R_i denotes the total revenue and $C(e_i)$ denotes the cost of effort. All standard assumptions regarding the cost function apply, that is, $C' > 0$ and $C'' > 0$. In particular, we assume $C(e_i) = \frac{\gamma}{2}e_i^2$ with $\gamma > 0$ such that the model has a closed form solution.

3.1 Characterization of the Equilibrium

When growers make decisions on how much effort to exert, the idiosyncratic productivity shocks u_i $(i = 1, ..., N)$ and the common productivity shock η have not yet been realized. Therefore, in this tournament game, ex ante, growers only differ in terms of their own ability and have the same information regarding other structural elements of the game. In such a case, a symmetric equilibrium is a natural outcome to analyze. The optimal strategy $e_i^* = s(\theta_i)$ is based on each grower's maximizing her ex-ante expected payoff with respect to e_i. After integrating out all the unknowns and assuming that all other growers adopt the same strategy $e_j^* = s(\theta_j)$ for $j \neq i$, the expected payoff function can be written as

$$\begin{aligned}
&E\pi_i = \int ... \int (R_i - C(e_i)) \prod_{j\neq i} g(\theta_j) \prod_{i=1}^N f(u_i)\, p(\eta) \prod_{j\neq i} d\theta_j \prod_{i=1}^N du_i d\eta \\
&= \int ... \int \left\{\left[a + b\left(\tfrac{1}{N}\sum_j f_j - f_i\right)\right] y_i - C(e_i)\right\} \\
&\prod_{j\neq i} g(\theta_j) \prod_{i=1}^N f(u_i)\, p(\eta) \prod_{j\neq i} d\theta_j \prod_{i=1}^N du_i d\eta \\
&= \int ... \int \left\{\left[a + b\left(\tfrac{1}{N}\sum_j \tfrac{1}{\theta_j e_j u_j \eta} - \tfrac{1}{\theta_i e_i u_i \eta}\right)\right] \theta_i e_i u_i \eta - C(e_i)\right\} \\
&\prod_{j\neq i} g(\theta_j) \prod_{i=1}^N f(u_i)\, p(\eta) \prod_{j\neq i} d\theta_j \prod_{i=1}^N du_i d\eta \\
&= \int ... \int \left\{a\theta_i e_i u_i \eta + b\tfrac{1-N}{N} + b\tfrac{1}{N}\sum_{j\neq i} \tfrac{\theta_i e_i u_i}{\theta_j e_j u_j} - C(e_i)\right\} \\
&\prod_{j\neq i} g(\theta_j) \prod_{i=1}^N f(u_i)\, p(\eta) \prod_{j\neq i} d\theta_j \prod_{i=1}^N du_i d\eta.
\end{aligned} \tag{6}$$

Now we are in the position to state the following result:

Proposition 1 *The symmetric pure-strategy Bayesian Nash equilibrium* $e_i^* = s(\theta_i)$ $(i = 1, ...N)$ *of this cardinal tournament game is*

$$e_i^* = s(\theta_i) = \theta_i \sqrt{\frac{a^2 E^2(\eta) E^2(u_i)}{4\gamma^2} + \frac{b(N-1)}{N\gamma} E(u_i) E\left(\frac{1}{u_i}\right) E\left(\frac{1}{\theta_i^2}\right)} \tag{7}$$

where $E(\cdot)$ *denotes the mean of the random variable in parenthesis.*

Proof. The first order condition for (6) with respect to e_i is[3]

$$\begin{aligned} &\int ... \int \left[a\theta_i u_i \eta + b\tfrac{1}{N} \textstyle\sum_{j \neq i} \tfrac{\theta_i u_i}{\theta_j e_j^*(\theta_j) u_j} - \gamma e_i^*\right] \\ &\textstyle\prod_{j \neq i} g(\theta_j) \prod_{i=1}^N f(u_i) p(\eta) \prod_{j \neq i} d\theta_j \prod_{i=1}^N du_i d\eta = 0. \end{aligned} \tag{8}$$

Using the independence assumption regarding θ_i, u_i and η, (8) can be written as

$$a\theta_i E(u_i) E(\eta) + \frac{b\theta_i (N-1)}{N} E(u_i) E\left(\frac{1}{u_i}\right) E\left(\frac{1}{\theta_j e_j^*(\theta_j)}\right) = \gamma e_i^* \tag{9}$$

where we have used the fact that $E\left(\frac{1}{u_j}\right) = E\left(\frac{1}{u_i}\right)$. Rearranging terms, we have

$$E\left(\frac{\gamma}{a\theta_i^2 E(u_i) E(\eta) + \frac{b\theta_i^2 (N-1)}{N} E(u_i) E\left(\frac{1}{u_i}\right) E\left(\frac{1}{\theta_j e_j^*(\theta_j)}\right)}\right) = E\left(\frac{1}{\theta_i e_i^*}\right). \tag{10}$$

Since we focus on a symmetric equilibrium strategy and the growers' abilities θ_i $(i = 1, ..., N)$ are integrated out, $E\left(\frac{1}{\theta_j e_j^*(\theta_j)}\right) = E\left(\frac{1}{\theta_i e_i^*}\right)$ for all $j \neq i$. After labeling this term as M, we obtain

$$E\left(\frac{\gamma}{a\theta_i^2 E(u_i) E(\eta) + \frac{b\theta_i^2 (N-1)}{N} E(u_i) E\left(\frac{1}{u_i}\right) M}\right) = M. \tag{11}$$

Rearranging, we get

$$\left[\frac{\gamma}{aE(u_i) E(\eta) + \frac{b(N-1)}{N} E(u_i) E\left(\frac{1}{u_i}\right) M}\right] E\left(\frac{1}{\theta_i^2}\right) = M, \tag{12}$$

which can be further written as a quadratic equation in M

$$\frac{aE(u_i) E(\eta)}{\gamma} M + \frac{b(N-1)}{N\gamma} E(u_i) E\left(\frac{1}{u_i}\right) M^2 - E\left(\frac{1}{\theta_i^2}\right) = 0. \tag{13}$$

[3] It is straightforward to show that the second order sufficient condition for maximization holds as well.

with a solution[4]

$$M = \frac{-aE(\eta) + \sqrt{a^2E^2(\eta) + \frac{4b(N-1)\gamma}{N}\frac{1}{E(u_i)}E\left(\frac{1}{u_i}\right)E\left(\frac{1}{\theta_i^2}\right)}}{\frac{2b(N-1)}{N}E\left(\frac{1}{u_i}\right)}. \tag{14}$$

Finally, plugging (14) into (9) completes the proof by obtaining the expression for optimal effort as

$$\begin{aligned}
e_i^* &= \frac{a\theta_i E(u_i)E(\eta)}{\gamma} + \frac{\theta_i}{\gamma}E(u_i)\left[\frac{-aE(\eta) + \sqrt{a^2E^2(\eta) + \frac{4b(N-1)\gamma}{N}\frac{1}{E(u_i)}E\left(\frac{1}{u_i}\right)E\left(\frac{1}{\theta_i^2}\right)}}{2}\right] \\
&= \theta_i\left\{\frac{aE(u_i)E(\eta)}{\gamma} - \frac{aE(u_i)E(\eta)}{\gamma} + \frac{E(u_i)\sqrt{a^2E^2(\eta) + \frac{4b(N-1)\gamma}{N}\frac{1}{E(u_i)}E\left(\frac{1}{u_i}\right)E\left(\frac{1}{\theta_i^2}\right)}}{2\gamma}\right\} \\
&= \theta_i\sqrt{\frac{a^2E^2(\eta)E^2(u_i)}{4\gamma^2} + \frac{b(N-1)}{N\gamma}E(u_i)E\left(\frac{1}{u_i}\right)E\left(\frac{1}{\theta_i^2}\right)}. \qquad (15)
\end{aligned}$$

■

3.2 Comparative Statics

With the closed form solution for optimal effort e_i^*, it is easy to study various comparative statics results. First, optimal effort e_i^* is increasing in ability θ_i and decreasing in the marginal cost of effort γ. Also, it is increasing in the base payment a and the slope of the bonus payment b. Furthermore, optimal effort is increasing with the number of tournament contestants N and the expectation of the common shock $E(\eta)$. The comparative static result with respect to $E(u_i)$ is ambiguous as $E\left(\frac{1}{u_i}\right)$ is a nonlinear function of $E(u_i)$. Finally, notice that

$$E\left(\frac{1}{\theta_i^2}\right) = E^2\left(\frac{1}{\theta_i}\right) + V\left(\frac{1}{\theta_i}\right) \tag{16}$$

where $V(\cdot)$ denotes the variance of the random variable in parenthesis. Therefore, for a constant $E\left(\frac{1}{\theta_i}\right)$, increasing $V\left(\frac{1}{\theta_i}\right)$, increases the optimal effort e_i^*. Since θ_i is defined as

[4] The other root is automatically ruled out because $M \geq 0$ by construction.

grower i's ability (efficiency) parameter, $\frac{1}{\theta_i}$ can be thought of as inaptitude parameter. This implies that for a given mean of the growers' inaptitude parameters, larger variance (more heterogenous growers) produces higher optimal effort. This means that any grower i, given her own inaptitude, knowing that the variance of other growers' inaptitudes is large will exert more effort. When contestants are highly heterogenous, i.e. the variance of the inaptitude parameters is large, it is more likely for her to draw a contestant with extremely large or extremely small inaptitude. In both cases she has more incentives to work hard. This is because if the competitor turns out to be very good and pushes the mean performance high, then grower i has to work hard to avoid losing too much. On the other hand, if the competitor turns out to be very bad and pushes the mean performance low, grower i again wants to work hard as she can easily reap big profits in cardinal tournament payment scheme.

4 Structural Estimation

As explained in detail in Section 2, our data set is an unbalanced panel where $\overline{N}$ growers that grow chickens for the same integrator compete in different tournaments of size $N < \overline{N}$. Denoting y_{kit} as the kth ($k = 1, ..., N_t$) observation in tournament t ($t = 1, ..., T$) that records grower i's ($i = 1, ...\overline{N}$) performance, we can rewrite (2) as

$$\begin{aligned} y_{kit} &= \theta_i e_{it} u_{kit} \eta_t \\ &= \theta_i^2 \sqrt{\frac{a^2 E^2(\eta_t) E^2(u_{kit})}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u_{kit}) E\left(\frac{1}{u_{kit}}\right) E\left(\frac{1}{\theta_i^2}\right)} u_{kit} \eta_t \end{aligned} \tag{17}$$

where the second equality follows from (7). Taking logarithms of both sides of (17) yields

$$\begin{aligned} \log y_{kit} &= 2\log\theta_i + 0.5\log\left[\frac{a^2 E^2(\eta_t) E^2(u_{kit})}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u_{kit}) E\left(\frac{1}{u_{kit}}\right) E\left(\frac{1}{\theta_i^2}\right)\right] \\ &\quad + \log u_{kit} + \log \eta_t \\ &= 2\log\theta_i + 0.5\log z_t + \log\eta_t + \log u_{kit} \end{aligned} \tag{18}$$

where $z_t = \left[\frac{a^2 E^2(\eta_t) E^2(u_{kit})}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u_{kit}) E\left(\frac{1}{u_{kit}}\right) E\left(\frac{1}{\theta_i^2}\right)\right]$ varies only across tournaments due to its dependence on N_t. If N_t is fixed across tournaments, then z_t is fixed across tournaments since other terms like $E\left(u_{kit}\right)$, $E\left(\frac{1}{u_{kit}}\right)$, $E\left(\frac{1}{\theta_i^2}\right)$ and $E\left(\eta_t\right)$ are all fixed constants. For estimation and identification purpose, we also assume that $E\left(\log u_{kit}\right)=0$ and $\log \eta_t$ is normally distributed with mean 0 and variance σ_η^2. The zero mean assumptions for $\log u_{kit}$ and $\log \eta_t$ can be regarded as normalization assumptions as both u_{kit} and η_t are productivity shocks.

Our estimation strategy consists of two steps. In the first step, with the assumption that $E\left(\log u_{kit}\right)=0$, we propose the following reduced-form regression

$$\log y_{kit}=\sum_{i=2}^{\overline{N}} \mu_i d_{kit}+\sum_{t=1}^{T} \lambda_t g_{kit}+\epsilon_{kit}. \tag{19}$$

Coefficients on grower dummies $\widehat{\mu}_i$ ($d_{kit}=1$ if the kth observation in tournament t records grower i's performance, $d_{kit}=0$ elsewhere) can be used as estimates of growers' abilities $2\log\theta_i$ from the output function (18). To avoid multi-collinearity we excluded the dummy variable for the first grower and hence the ability of the first grower, $\theta_1=1$, or $\log\theta_1=0$. Also, the coefficients $\widehat{\lambda}_t$ associated with tournament dummies g_{kit} can be used to estimate the sum of the deterministic part of the output function (18) that only varies over tournaments and the common shock $\left(0.5\log z_t+\log\eta_t\right)$. Finally, the estimated residual term $\widehat{\epsilon}_{kit}$ can be used as an estimate of the idiosyncratic productivity shock $\log u_{kit}$.

In the second step, we exploit the following relationship

$$\lambda_t=0.5\log z_t+\log\eta_t \tag{20}$$

where $z_t=\left[\frac{a^2E^2(\eta_t)E^2(u_{kit})}{4\gamma^2}+\frac{b(N_t-1)}{N_t\gamma}E\left(u_{kit}\right)E\left(\frac{1}{u_{kit}}\right)E\left(\frac{1}{\theta_i^2}\right)\right]$. From the first step we obtained an estimate for λ_t, and with the estimated $\widehat{\epsilon}_{kit}$, we can obtain an estimate $\widehat{u}_{kit}=\exp\left(\widehat{\epsilon}_{kit}\right)$. Furthermore, $\exp\left(\frac{\widehat{\mu}_i}{2}\right)$ can be used as an estimate for θ_i. With $\widehat{u}_{kit}$ and $\widehat{\theta}_i$, estimates for $E\left(u_{kit}\right)$, $E\left(\frac{1}{u_{kit}}\right)$ and $E\left(\frac{1}{\theta_i^2}\right)$ can be easily constructed. Next, note that we assume $\log\eta_t$ is normally distributed with mean 0 and variance σ_η^2. This assumption leads to the result that $E\left(\eta_t\right)=\exp\left(\frac{\sigma_\eta^2}{2}\right)$. As a result, the only two unknowns in (20) are γ and σ_η^2 and we apply MLE to (20) to obtain estimates for these two unknown parameters. As this step of estimation uses variables generated from results of the first step estimation, standard errors of the second step will be obtained using the bootstrap method. This completes the structural estimation of the model.

4.1 Estimation Results

In the first step, we run the simple OLS regression of (19). The R^2 is 0.9408. After estimation, we can recover $\widehat{\mu}_i$ (hence $\widehat{\theta}_i$) for each grower, $\widehat{\lambda}_t$ for each tournament and $\widehat{\epsilon}_{kit}$ (hence $\widehat{u}_{kit}$) for each observation. These results, together with the summary statistics for the dependent variable used in estimation, $\log y_{kit}$, are reported in Table I.

Using the results from the first step, we estimate $E(u_{kit})$ using $\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\widehat{u}_{kit}$, $E\left(\frac{1}{u_{kit}}\right)$ using $\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\frac{1}{\widehat{u}_{kit}}$, and $E\left(\frac{1}{\theta_i^2}\right)$ using $\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\widehat{\theta}_i^2}$. In the second step, we maximize the following likelihood function

$$L = \sum_{t=1}^{T}\phi\left(\frac{\log\widehat{\lambda}_t - 0.5\log\widehat{z}_t}{\sigma_\eta}\right) \tag{21}$$

where $\widehat{\lambda}_t$ is taken directly from the first step estimation results,

$$\widehat{z}_t = \left[\frac{a^2\exp\left(\sigma_\eta^2\right)\left[\widehat{E}\left(u_{kit}\right)\right]^2}{4\gamma^2} + \frac{b\left(N_t-1\right)}{N_t\gamma}\widehat{E}\left(u_{kit}\right)\widehat{E}\left(\frac{1}{u_{kit}}\right)\widehat{E}\left(\frac{1}{\theta_i^2}\right)\right] \tag{22}$$

and ϕ is the standard normal density.[5] Estimation results are collected in Table II.

As explained before, both the regressand and the regressors in the second step are constructed using results from the first step estimation. Consequently, standard errors in the second step estimation will be biased. To obtain the correct standard errors, we use the bootstrap method. Furthermore, to preserve the unbalanced panel structure of the data set, we use the nonparametric residual bootstrap method (e.g. Wooldridge 2002, pp. 380). The exact procedure can be detailed as follows. From the first step estimation, we recover the residuals $\widehat{\epsilon}_{kit}$ for each observation. Then, at each iteration of the bootstrap, we re-sample

[5] In the data, the unit used to calculate the payment is 0.1 cent and a is set at 30 (3 cents) and b is set at 1. In estimation, a is set at 3 and b is set at 0.1 such that the welfare results below are in terms of cents instead of 0.1 cents.

with replacement from the recovered residuals to obtain a new sample of residuals. The new sample of residuals are added back to $\sum_{i=2}^{N} \widehat{\mu}_i d_{kit} + \sum_{k=1}^{T} \widehat{\lambda}_t g_{kit}$ to obtain a new sample of $\log y_{kit}$. Finally, the entire estimation (both the first step and the second step) is repeated using the new sample of data. We repeat this procedure 200 times. The bootstrap standard deviation of the second step parameters estimates are used as their standard errors.

Next, using the structural estimates, we are able to compute all quantities of economic interest. The results are collected in Table III. We find that on average, a grower exerts 0.0325 unit of effort per \$1 worth of inputs and the cost associated with this effort level is 0.0586 cents. As a result, on average, a grower earns 0.0971 cents in total payment and her profit amounts to 0.0385 cents per \$1 worth of input.

5 Counterfactual Analysis

The most interesting theoretical result obtained in Section 3 shows that for a fixed $E\left(\frac{1}{\theta_i}\right)$ (the mean of the grower inefficiency parameters), an increase in $V\left(\frac{1}{\theta_i}\right)$ (the variance of the grower inefficiency parameters) generates an increase in the equilibrium effort e_i^*. This implies that the principal can gain from heterogenizing the tournament groups as long as he can sell the product at a price higher than the payment to the contract growers and as long as he does not violate growers' participation constraint. On the other hand, the effect of this policy change on growers' welfare is unclear. This is because higher effort leads both to higher productivity and hence higher payment, but also increase the cost of effort. One advantage of the structural econometrics approach is that it enables us to use the model estimates to quantify the welfare effects of such regime shifts or policy proposals.

To quantify these welfare effects, we run a counterfactual experiment by increasing $E\left(\frac{1}{\theta_i^2}\right)$ by 10%, holding $E\left(\frac{1}{\theta_i}\right)$ fixed. Then, using the structural estimates and holding the productivity shocks at the level before the policy change, we can first compute the new optimal effort level for each grower, and subsequently compute their outputs, payments, and profits under the new scenario. The results from such an experiment are collected in Table IV. We find that with the more heterogenous groups, growers' equilibrium efforts (and hence their outputs and payments), would increase by 4.06% on average. On the other hand, growers' costs of effort would increase on average by 8.28%, and as a result, on average, their profits would decrease by about 2.92%. More specifically, the percentage changes in growers' profits range from -22.78% to 0.55% and only in 31 out of 3274 observations the positive profit change is recorded. The results clearly show that if the principal makes the tournaments more heterogenous, the average contract grower effort, output and payment would go up but the majority of growers would be worse off than before the change.

As mentioned briefly before, worsening of the anticipated profits may violate some growers' participation constraint causing them to drop out of the contract by not signing the new contract when contracts are up for renewal. This problem can be retroactively fixed by keeping the growers *ex-post* at the same level of profits (utility) as before the change by paying them a lump-sum payment equal to the welfare loss caused by the proposed change. This will also enable us to quantify the effect of heterogenizing the tournament groups on social welfare (the sum of the principal's and growers' profits). Based on the result from Table IV, we see that for 1 cent worth of inputs, on average, a grower's output increases from 0.0323 pounds of chicken to 0.0337 pounds. Assuming the industry is perfectly competitive, the principal can sell all his output at the prevailing market price. Therefore, the principal gets an additional revenue of 0.0823 cents by selling the additional 0.0014 pounds of chicken meat.[6] On the other side, on average, a grower's payment decreases from 0.0385 cents to 0.0376 cents, for a 0.0009 cents loss. If the principal pays back this amount as a lump sum transfer to the growers, he would still end up better off than before the change. Consequently, the net social welfare gain associated with this policy change is positive. Heterogenizing tournament groups by 10% results in 0.0814 cents (per 1 cent worth of inputs) increase in social surplus. On a per grower basis, the average settlement cost per grower in the data is 31.26 cents per pound of chicken produced and the average number of pounds

[6] The 12-city broiler price average for the period covered by our data set (July 1995 - July 1997) is 58.78 cents per pound.

produced per grower is 240,390 pounds, implying an average value of inputs (feed and chicks and other chargeable costs) per grower at \$75145.91. Therefore, the increase in social welfare equals \$6116.88 (0.0814 times \$75145.91) on a per grower basis.

6 Conclusion

The motivation for writing this paper started by observing that poultry integrators generally do not attempt to place their contract growers into more homogenous groups in which they compete against each other for the bonuses determined by the individual grower's production cost reduction relative to the group average cost. This fact was even more puzzling knowing that via repeated contracting with the same pool of growers for a long period of time, the integrators know a lot about their growers' idiosyncrasies, and can recognize good growers from bad growers, yet they never exploit this information to their advantage.[7] This observation was combined with the received theory showing that mixing players of different abilities in rank-order tournaments produce suboptimal results. Yet, we did not realize that these theoretical results do not extend into cardinal tournament games.

Our theoretical result shows that there is actually a very good reason for why poultry integrators never attempt to homogenize the groups of growers they place in separate tournaments. The actual assembly of tournament groups seems to be solely guided by sequencing and logistics of the production process. Our most interesting theoretical result shows that for a fixed mean of the grower inefficiency parameters, an increase in the variance of the grower inefficiency parameters generates an increase in the equilibrium effort. This implies that the principal can actually gain from heterogenizing the tournament groups. On the other hand, the effect of this change on growers' welfare is unclear because higher effort leads to higher productivity and hence higher payment, but also increases the cost of effort. Our counterfactual analysis shows that under reasonable assumptions the integrator's gain is actually larger

[7] The random composition of tournament groups in broiler contracts was empirically confirmed by Levy and Vukina, 2004.

than the growers' losses and that he can in principle compensate growers and still be better off. The results suggest that heterogenizing groups in cardinal tournaments, to the extent that it is practicable, may be efficient in the sense of increasing social surplus.

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Table I: Estimates from the First Stage Estimation

	Mean	Standard Deviation	Number of Observations
$\log y_{kit}$	-3.4369	0.1040	3247
$\widehat{\mu}_i$	-0.0138	0.0570	356
$\widehat{\lambda}_t$	-3.4196	0.0988	104
$\widehat{\epsilon}_{kit}$	-4.1227*10^{-14}	0.0253	3247
$\widehat{\theta}_i$	0.9918	0.0092	104
$\widehat{u}_{kit}$	1.0003	0.0247	3247

Table II: Estimates from the Second Stage Estimation[8]

	Mean	Standard Error	t-Stat
σ_η^2	0.0097	9.1527*10^{-5}	106.36
γ	111.0021	1.0068	110.25
Log Likelihood	188.8670		

Table III: Quantities of Economic Interest

	Mean	Standard Deviation
R_{kit} (cents)	0.0971	0.0114
e_{it}	0.0325	0.0003
$C(e_{it})$ (cents)	0.0586	0.0010
$R_{kit} - C(e_{it})$ (cents)	0.0385	0.0112

[8]Standard errors are based on 200 iterations of boostrap

Table IV: Effect of Heterogenizing Tournament Groups

	Before		After		Change	
	Mean	Stan. Dev.	Mean	Stan. Dev.	Mean	Stan. Dev.
y_{kit}	0.0323	0.0034	0.0337	0.0035	4.06%	0.41%
R_{kit} (cents)	0.0971	0.0114	0.1011	0.0118	4.06%	0.20%
e_{it}	0.0325	0.0003	0.0338	0.0003	4.06%	0.41%
$C(e_{it})$ (cents)	0.0586	0.0010	0.0634	0.0011	8.28%	0.85%
$R_{kit} - C(e_{it})$ (cents)	0.0385	0.0112	0.0376	0.0116	-2.92%	2.10%