Time-varying Yield Distributions and the Implications for Crop Insurance Pricing

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Abstract

The objective of this study is to evaluate and model the yield risk associated with corn production. This study focuses on the risk of yield variability and addresses changes in the parameters of the yield distribution that are allowed to vary with time. The conventional approach is based on a two-stage method in which the yield is first detrended and then the estimated residuals are modeled by using various distributions. We propose a model that allows the simultaneous estimation of the trend and error (residual) distributions. Several model selection tools (information criterion, cross-validation, etc.) are used to choose the suitability of the proposed models for the yield data obtained from the Adair county of Iowa. In a simulation study, the two alternative yield modeling procedures are used to price an insurance contract that covers the yield loss triggered on a county level yield coverage.

Key Words: Crop Insurance, Model Comparison, Time-varying Distribution

1 Introduction

Agricultural risk is typically assumed to originate from the unanticipated movements in prices and yields. Managing agricultural production risk is important for corn farmers since they may be exposed to various sources of risk in corn production. From the time of planting corn in February to the harvest and selling season in November, the expected price and yield can change immensely. Most of this risk comes in the form of catastrophic events, such as hurricanes, drought or excess rainfall, which cause yields to be substantially below normal. The risk can also come from large swings in corn prices.

Farmers typically consider the USDA facilitated and subsidized Federal Crop Insurance program as a financial management tool. The Federal Crop Insurance Program (FCIP) is a useful tool to protect farmers from risk. Insurance against poor crop yields has been available for many years. Yield based insurance policies include individual farm-level Multiple Peril Plan (MPCI) and county-level Group Risk Plan (GRP). The focus in this study will be on evaluating and modeling the yield risk associated with county yield shortfalls (the Group Risk Plan (GRP)). Accurate premium rates are crucial for an actuarially sound insurance program, which needs accurately modeled crop yield distributions in order to precisely estimate crop yield risk and for the proper design and rating of crop insurance contracts. In most empirical analyses, the only conditioning variate used to explain the mean yields is time. This study restricts attention to modeling yield densities conditional on the temporal process of yields. Historical data on annually-averaged corn yields suggest a strong upward trend over time. Advances in biotechnology and the development of new hybrids may significantly affect the distributions of corn yields. In particular, various moments of the distributions may evolve over time as the technology progresses. These changes can complicate efforts to accurately model yield distributions using data observed over time.

Many crop insurance studies have been done to determine the distributional model that best characterizes crop yields. These studies have indicated that agricultural yields can be modeled in different ways. An adequate representation of yields risk requires an estimate of the probability density function (pdf) so that all moments of the distribution, such as the mean, variance and skewness, can be characterized. The modeling approach varies from parametric models to non-parametric (Goodwin and Ker (1998)) models.

Usually, when crop yield data are used to fit models, the trend component is estimated before assessing the distribution of the yield data—generally in an ad-hoc manner. The typical approach in the literature has been to first detrend the time series of yield data and then estimate the yield distribution using the detrended yield data as if they are "observed" data. These approaches are based on two stages. The first stage fits the trend to the data and the second models the detrended yield data.

This two-stage method of estimating yield distribution most likely fails to adequately capture the uncertainty of the trend estimation in the first stage. Some accuracy will be lost when estimating the distribution of the residuals derived from the yield trends. The estimated yields tend to be underestimated due to some extreme low yields in catastrophic events which drive the trend regression curve down. The choice of detrending procedure may induce invalid inferences about the random component of the crop yield model. Likewise, these approaches may not be efficient as the uncertainty of the first-stage estimation is ignored, which results in under-estimation of the true uncertainty.

This study uses alternative methods to measure conditional yield risk and calculate premium rates for crop insurance contracts in a more accurate and systematic way. This study proposes a method to simultaneously model the time trend and the parameters of the yield distribution by directly using the actual yield data instead of detrending the yields prior to modeling the distribution on a two-step basis. The time-varying Beta family of distributions are proposed for the crop yield data in this study. The method of maximum likelihood estimator (MLE) is obtained for different time trend structural models of the crop yields. This method essentially models the trend of all moments of the distribution simultaneously by allowing shape, scale, and location parameters of the specific distribution family to vary over time. Specific functional forms are constructed to ensure that parameters estimates are of the required sign and magnitude.

Based on this time-varying distribution estimation approach, the rest of the paper is organized as follows. Section 2 provides some background and reviews the literature on modeling risk in yields. Section 3 outlines the conventional two-stage approach of modeling yield distributions. Section 4 explores the time-varying approach for econometric modeling of yield distributions. Section 5 discusses some model comparison results. A simulation study is conducted in Section 6 to compute the insurance premium rate of these models. A method of "cross-rate-validation" is proposed to calculate the prediction error in term of premium rates based on these two models. The conclusion is drawn that the time-varying model outperforms the conventional model in terms of both out-of-sample prediction power and the predicted premium rate errors.

2 Background on Modeling Yield Risk and Literature Review

This section is intended to offer some background on risks in agricultural production and the crop insurance program, with a focus on the protection against risks in yield shortfalls. The current literature on modeling yield risk in crop insurance will be reviewed.

Federally regulated crop insurance programs have become a prominent part of U.S. agricultural policy. A variety of crop yield insurance plans are available.[1] Standard crop yield insurance, termed Multiple Peril Crop Insurance, pays an indemnity at a predetermined price to replace yield losses. Yield insurance includes the Actual Production History plan (APH), which allows a farmer to insure yield based on their proven yield history. This plan's guarantees are based on individual insurance units within a farm's Losses. "Group-risk" yield insurance, termed as the Group Risk Plan (GRP), is based on the county's average yield. Insured farmers collect an indemnity when the county average yield falls beneath a yield guarantee, regardless of the farmer's actual yields. Since county yield is typically less variable than the farm yield, more coverage is allowed under GRP than under the APH plan.

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[1] The crop insurance programs reviewed in this thesis are the crop policies available on the website of Risk Management Agency (RMA) in United State Department of Agriculture (USDA).

Measurement of yield risk requires the ability to model the distribution of yields, which are useful to design and rate crop insurance contracts. Loss probabilities are the probabilities that yields below (or above) some threshold will be observed, which is given by the area under the density to the left of the guaranteed yield. Suppose an insurance contract will insure some proportion ($\lambda \in (0,1)$) of the expected crop yield (y^e). If $y < \lambda y^e$, the insurer will pay ($\lambda y^e - y$) as an indemnity. An actuarially fair premium of a GRP contract is equal to the expected loss of this contract. The expected loss (in bushels) for this insurance contract that guarantees $\lambda \times 100\%$ of the predicted yield (y^e) takes the form of

$$\mathrm{E(Loss)} = E[(\lambda y^e - y)I(y \leq \lambda y^e)] = E[(\lambda y^e - y)^+]$$

where $(\lambda y^e - y)^+ = max(0, \lambda y^e - y)$. y denotes the observed annual county level yield and y^e represents the predicted guaranteed yield. To calculate the expected loss requires the estimation of the distribution of yields. Various crop modeling approaches are proposed in the agricultural economics literature focusing on investigating these risk factors associated with crop production and studying how risk should be characterized and measured. An adequate representation of risk requires an estimate of the probability density function so that all relevant moments of the distribution, such as mean, variance and skewness, can be characterized. The modeling approach ranges from non-parametric (Goodwin and Ker, 1998) to parametric (Chen and Miranda, 2004) methods. The parametric approach of modeling yields usually involves selection of candidate distributions, parameter estimation and assessment of goodness-of-fit. The popular candidates used in the agricultural economics literature include: the Beta distribution (Nelson, 1990), the Weibull distribution (Chen and Miranda, 2004) and the Logistic distribution (Sherrick et al., 2004).

3 Conventional Two-stage Estimation Framework

Previous work on goodness of fit results for Normal, Beta and Weibull distributions has shown that the Beta distribution usually fits county-level corn yield data well. This section will estimate the Beta distribution of yield in a conventional two-stage estimation framework. The data used in this analysis include corn and soybean yield data for Iowa from 1927 to 2006 obtained from the National Agricultural Statistics Service (NASS).

3.1 Econometric Model — The Standard Two-stage Estimation Method

The conventional two-stage estimation model states deviations of yield from its mean (which is captured by a time trend) as a Beta distribution, and is thus referred to as the "detrended BETA model" (Model I). A deterministic trend instead of a stochastic trend is usually used to capture the development of the yields. The main justification for using a deterministic component, as pointed out by Just and Weninger (1999), is that if economic variables move slowly through time, then approximation of a deterministic component may be sufficient to model a yield distribution. Before assessing the distribution of the yields, the trend component is controlled for in this two-stage estimation model. The usual approach to remove deterministic factors in yield is to use detrending regressions to fit a quadratic trend model in an ad-hoc manner as shown in the following equation.

$$y_t = \alpha + \beta_1 \tilde{t} + \beta_2 \tilde{t^2} + e_t.$$

where y_t is the observed crop yield data in year t, and $t = 1, 2, ..., 80$ stands for $year = 1927, 1928, ..., 2006$. The time variable in the above regression equation is rescaled as $\tilde{t} = t/100$. OLS parameter estimates are obtained from this regression and are shown in Table 1.

After regressing the yields on a quadratic (or linear) time trend, the residuals $\hat{e}_t = y_t - \hat{y}_t$ and the predicted yield for 2006, $\hat{y}_{2006}$ can be calculated, where $\hat{y}_t = \hat{\alpha} + \hat{\beta}_1 \tilde{t} + \hat{\beta}_2 \tilde{t^2}$. From the residual plot as shown in figure 2, there is a positive trend in the variance when the yield increases. The Q-Q plots suggests that the residual is more negatively skewed than

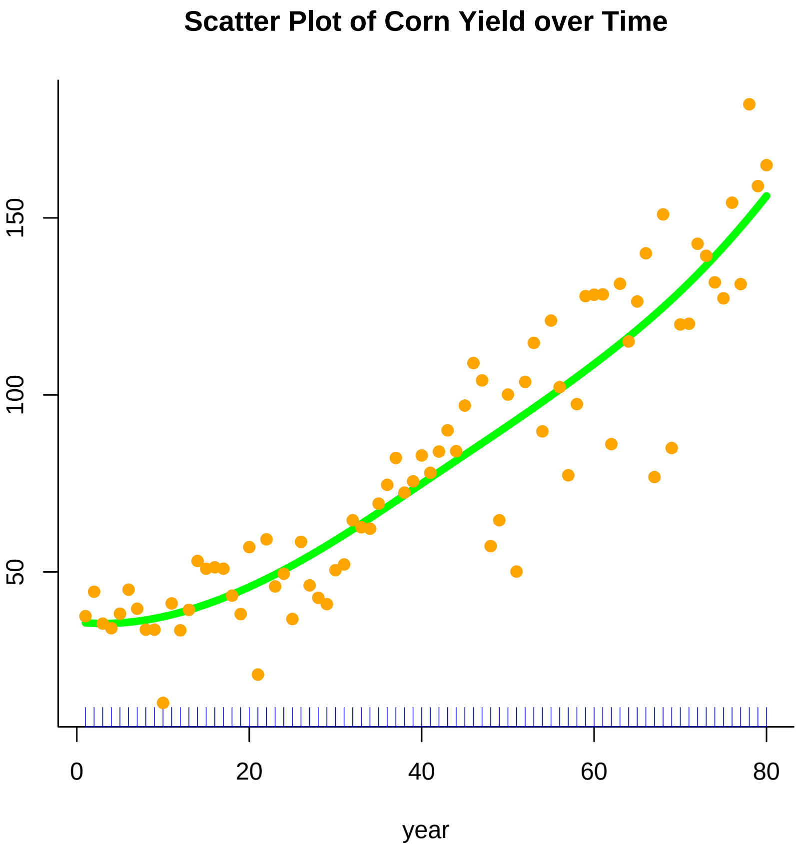

Figure 1: Scatter Plot of the corn yield over time, Adair County, Iowa.

the normal distribution, which suggests that a Beta distribution might be a good candidate fit. The Goodness-of-Fit tests for Beta distribution report a Chi-square statistic equal to 2.67 with p-value of 0.62, which suggests that a Beta distribution provides a good fit. From the data properties explored above, assuming that errors are proportional to the predicted mean, the proportional adjustment can be applied to obtain the detrended yield shocks. This approach is ad hoc but it is common in practice. The normalized and detrended data $\tilde{y}_t$ are $\tilde{y}_t = \hat{y}_{2006}(1 + \frac{\hat{e}_t}{\hat{y}_t})$.

Suppose that the normalized yield is a static Beta distributed random variate $\tilde{y}_t \sim Beta(\alpha, \beta, \theta, \delta)$. This implies that actual yield $y_t \sim Beta(\alpha, \beta, \tilde{\theta}_t, \tilde{\delta}_t)$, where $\tilde{\theta}_t = \rho_t \theta, \tilde{\delta}_t = \rho_t \delta$ with $\rho_t = \frac{\hat{y}_t}{\hat{y}_T}$. In this case, the actual yield distribution is a Beta distribution with constant shape parameters and time-varying location and scale parameters. The log-likelihood function of general Beta distribution with two shape parameters α, β and location θ and scale δ

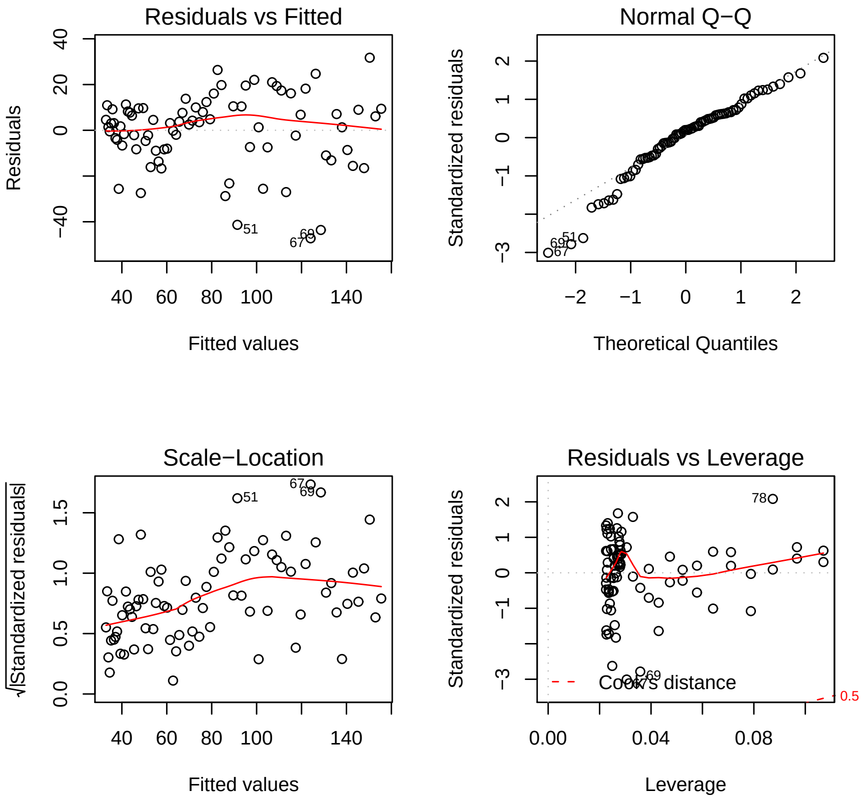

Figure 2: OLS Residual Plot of Annual Corn Yield, Adair County, Iowa

parameter is shown in the equation below.

$$\text{LLF}(\alpha,\beta,\theta,\delta|\tilde{y}_t, t=1,2,...,T) = (\alpha-1)\sum_{t=1}^{T} log(y_t-\theta) + (\beta-1)\sum_{t=1}^{T} log(\delta+\theta-y_t) - \\ Tlog(B(\alpha,\beta)) - T(\alpha+\beta-1)log(\delta)$$

where $log(B(\alpha,\beta)) = log(\Gamma(\alpha)) - log(\Gamma(\beta)) + log(\Gamma(\alpha+\beta))$.

3.2 Estimation Results

The parameter estimates of the normalized yield $\tilde{y}_t$ for this model $Beta(\alpha,\beta,\theta,\delta)$ can be obtained by maximizing $\text{LLF}(\alpha,\beta,\theta,\delta|\tilde{y}_t, t=1,2,...,T)$. Consequently, the MLE result of $Beta(\alpha,\beta,\tilde{\theta}_t,\tilde{\delta}_t)$ for the actual yield can be obtained. These results can be used to compare to the results from the time-varying yield distribution model proposed in the next section. The MLE results of detrended yield $\tilde{y}_t$ are reported in table 2 and table 3.

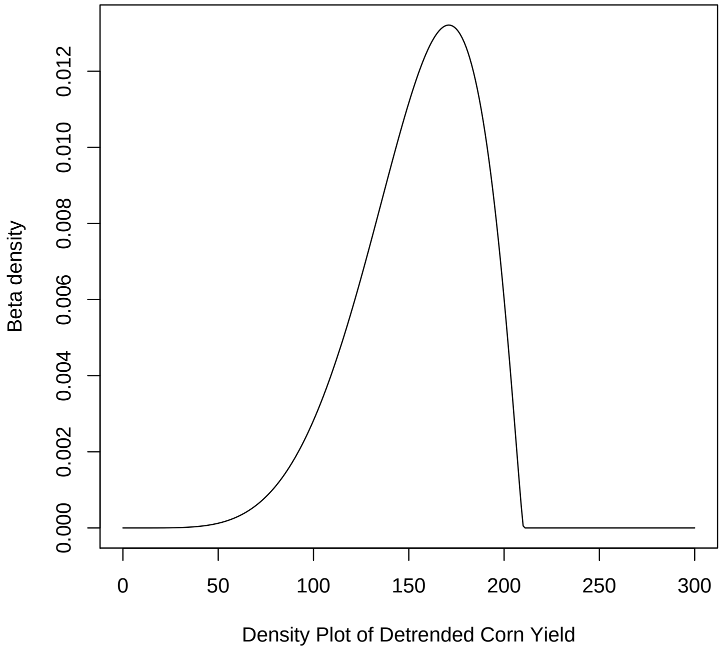

Figure 3: Detrended Beta Density Plot

The location parameter is not significant different from zero which is expected as the fit is based on detrended data. A reasonable assumption for the yield distribution is that the minimum possible yield is zero. A 3-parameter Beta family could be estimated by constraining the location parameter of the Beta distribution to be zero.

This two-stage method of estimating yield distribution is based on detrended yield data. The inferences about the risk of the crop yield depend on validity of the detrend method which connects the detrended yield back to actual yields. The choice of detrending procedure might induce invalid inferences about the random component of the crop yield model. The detrending method used here is ad hoc and may fail to adequately capture the uncertainty of the trend estimation in the first stage. Some accuracy will be lost when estimating the distribution of the residuals derived from the yield trends. As a result, these approaches may not be efficient as the uncertainty of the first-stage estimation is lost which results in under-estimation of the true uncertainty. A more accurate and systematic measurement method are needed to capture the conditional yield risk.

4 A Time-varying Yield Distribution Framework

In this section, a time-varying distribution approach is developed where the parameters of the Beta distribution and the time trend parameters are estimated simultaneously. Several time-varying Beta distribution models are examined to measure conditional yield risk in order to calculate premium rates for crop insurance contracts in a more accurate and systematic way. This represents an essential difference from previous approaches in the literature where the trend of yield data and the distribution of the normalized yield are estimated separately in two consecutive steps. The conventional two-stage approach can produce inadequate measures of uncertainty. The time-varying model accounts for the parameter uncertainty by simultaneously estimating the whole likelihood function of time-trend parameters and the distribution parameters in one step. The results of this application are compared to the results based on the conventional two-stage approach described in section 3.

4.1 Econometric Model

Different from the conventional model which estimates the yield distribution using the detended yield data as if they are "observed" data, the uncertainty of the parameters and the time trend are estimated in a whole likelihood function in the time-varying model. This modeling strategy accounts for the complexities of capturing the accuracy of yield distributions and characterizes the probabilistic models of the temporal process of yields involved with crop production. The model is estimated by maximizing the log-likelihood function of a Beta distribution which includes time-varying shape, location and scale parameters. These time varying parameters evolve with time in an exponential form, as discussed in section

4.1.1 and 4.1.2. The log-likelihood of the time-varying Beta distribution is as follows.

$$\begin{aligned}\text{LLF}(\text{b}|y_t, t=1,2,...,T) = (\alpha_t - 1)\sum_{t=1}^{T}\log(y_t - \theta_t) + (\beta_t - 1)\sum_{t=1}^{T}\log(\delta_t + \theta_t - y_t) - \\ T\log(B(\alpha_t, \beta_t)) - T(\alpha_t + \beta_t - 1)\log(\delta_t)\end{aligned}$$

where $log(B(\alpha_t, \beta_t)) = log(\Gamma(\alpha_t)) - log(\Gamma(\beta_t)) + log(\Gamma(\alpha_t + \beta_t))$ with $\alpha_t, \beta_t, \theta_t, \delta_t$ being functions of time and b is a vector of the time trend parameters. This study applies two different time trend structures to the parameters of the yield distributions and obtains estimates of the time-varying parameters for Beta distributions. The first structural model assumes an exponential time trend structure. This model assumes that the Beta shape parameters, location parameter and scale parameter are a quadratic function of time in an exponential functional form. The second structural model contains a linear time trend structure. This model assumes that the Beta shape, location and scale parameters are quadratic, linear-in parameters functions of time. The time-varying parameters estimates of these structural models are obtained by MLE. The functional forms of the time-varying parameters are discussed in the following two subsections.

4.1.1 Time-varying Beta with an exponential time trend structure—Model II

The time structure of these time-varying parameters in the Beta distribution can be captured in an exponential function form to guarantee the non-negativeness of the Beta parameters.

Let b_i,$i = 1, 2, 4, 5, 7, 8, 10, 11$ be the trend parameters of the time-varying Beta parameters, the four Beta parameters in each year t is as follows,[2] where $t = 1, 2, \ldots, 80$ stands for

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[2] The time in the above regression equation is rescaled as $\tilde{t} = \frac{t}{100}$.

$year = 1927, 1928, \ldots, 2006.$

$$\begin{aligned} \alpha_t &= exp(b_1 + b_2\tilde{t}), \\ \beta_t &= exp(b_4 + b_5\tilde{t}), \\ \theta_t &= exp(b_7 + b_8\tilde{t}), \\ \delta_t &= exp(b_{10} + b_{11}\tilde{t}) \end{aligned}$$

4.1.2 Time-varying Beta with an exponential quadratic time trend structure—Model III

To allow a more flexible time structure, the following functional forms are defined. The parameters b_3, b_6, b_9, b_{12} are coefficients on quadratic time, which allows for flexible change of the Beta parameters over time. Model II is nested in model III with parameters b_3, b_6, b_9, b_{12} equal to zero. A likelihood ratio test can be used to compare the usefulness of quadratic time parameters b_3, b_6, b_9, b_{12} as discussed in the following estimation section.

$$\alpha_t = exp(b_1 + b_2\tilde{t} + b_3\tilde{t}^2),$$

$$\beta_t = exp(b_4 + b_5\tilde{t} + b_6\tilde{t}^2),$$

$$\theta_t = exp(b_7 + b_8\tilde{t} + b_9\tilde{t}^2)$$

$$\delta_t = exp(b_{10} + b_{11}\tilde{t} + b_{12}\tilde{t}^2)$$

4.2 Estimation Results

By maximizing the LLF(y_t; b), the MLE estimate for $(b_1, b_2, b_4, b_5, b_7, b_8, b_{10}, b_{11})$ of this time-varying beta distribution is shown in table 4 with log-likelihood value equal to -328.6832. After substituting the MLE estimates of these time trend parameter b back into $\alpha_t, \beta_t, \theta_t, \delta_t$, the time-varying parameter estimates of Beta are obtained and all the moments of the distribution can be obtained in each year as shown in Figure 4.

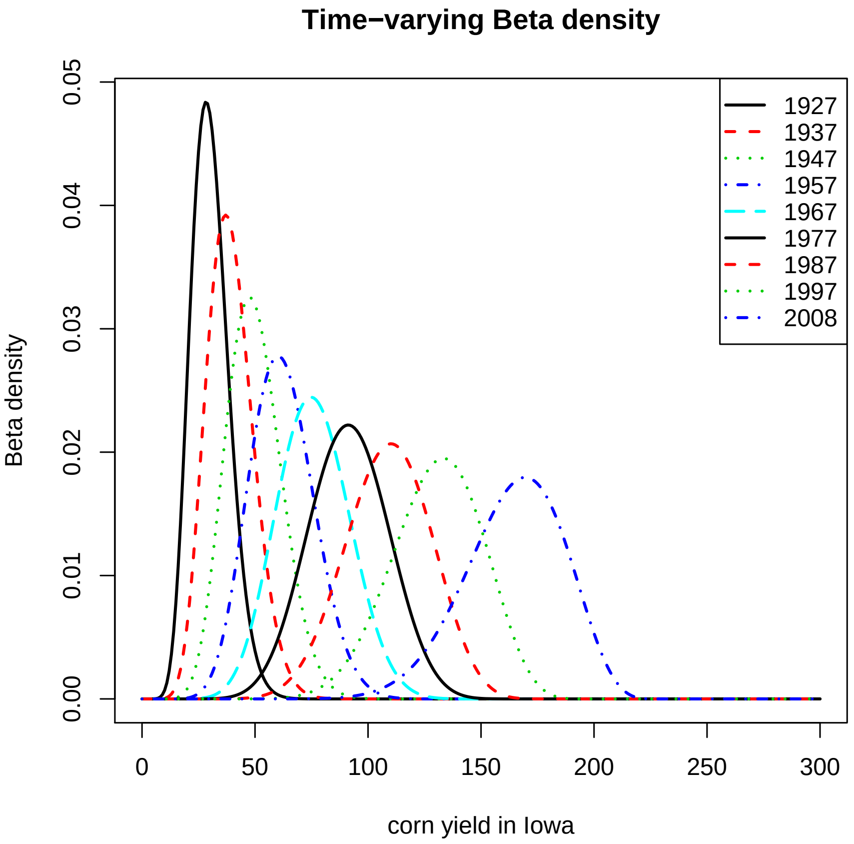

Figure 4: Time-varying Beta Density

The maximum likelihood estimate for $(b_1, \ldots, b_{12})$ of this time-varying beta distribution of model III is shown in table 4 with likelihood equal to -326.6221. A likelihood ratio test can be used to compare the model II and model III (which nests model II). The null hypothesis is $H_0 : b_3 = b_6 = b_9 = b_{12} = 0$. The LRT test statistics is $-2(L1 - L2) = 4.12$ with $\chi^2_{(4df)}$ distribution. This result shows that we do not reject the null hypothesis which means that the quadratic terms in model III are not necessary.

5 Model Comparison

This section provides insight into which model performs best by considering some desirable methods of model selection. The conventional two-stage model is parsimonious but may underfit the data and result in high bias. The model of a time-varying Beta distribution has more parameters which may overfit the data and yields high variance. A good model requires a balance between bias and variance. Some criteria and tests will help select the best model by searching through various candidate models.

5.1 AIC and BIC

The models discussed in Sections 3 and 4 can be compared by the Akaike Information Criterion (AIC) and the Bayesian Information Criterion of Schwarz (BIC). The idea of AIC (Akaike Information Criterion) is to maximize the "goodness of fit" minus "complexity". The expression of AIC is $-2l(\theta) + 2K$; the BIC is $BIC = -2l(\theta) + \mathrm{k}ln(T)$ where $l(\theta)$ is the log-likelihood of the model evaluated at the MLE, T is the number of observations in the sample and K is the number of parameters in the model. The smallest AIC or BIC gives the best model. Table 5 shows that model II has the lowest AIC and BIC, which indicates that model II is the most parsimonious and optimal model among the set of models that we have considered in this article. Moreover, $\Delta AIC(\Delta AIC = AIC - min(AIC))$ and $\Delta BIC(\Delta BIC = BIC - min(BIC))$ are significantly large for the conventional model, which are also evidence in support of the time-varying model.

5.2 Out-of-Sample Performance

An alternative model selection test is considered in order to be more consistent with the purpose of crop yield distribution estimates in the crop insurance. In this alternative approach, models are ranked based on their out-of-sample performance, especially two-years ahead or forward out-of-sample performance. The out-of-sample prediction test of a detrended Beta and the time-varying Beta is conducted to see which model has better out-of-sample performance, as we are usually interested in extrapolating into the future. In the practice of rating crop yield insurance, the yields are usually predicted two years out. According to this practice, a two-year-ahead out-of-sample test is done for each model. For example, to predict 1993's yield, the estimates are based on the sample from 1927 to 1991; to predict 1994's yield, the estimates are based on the sample from 1927 to 1992. The 2-year-ahead predict errors are recorded each year by moving forward to predict yields from 1993 up to 2006 and the average of this two-years ahead predict error ($Error_{2-ahead-pred}$) is as follows.

$$Error_{2-ahead-pred} = \frac{1}{14}\sum_{t=67}^{80}(\hat{y}_t - y_t)^2$$

where n is equal to the number of prediction periods.

An alternative out-of-sample test is to partition the sample into two parts and estimate based on the first part of the data, say the first 70 observations from 1927 to 1996. After the MLE estimates of the model are obtained, they are used to predict the yield in the second part of the sample—from 1997 to 2006. The sum of the squared difference between predicted value and the actual yield value are calculated as leave-10-out forecast error. $Error_{leave10out} = \frac{1}{10}\sum_{t=71}^{80}(\hat{y}_t - y_t)^2$. Table 6 shows that the predicted errors of the time-varying Beta model is 20% lower than that of the detrended Beta model.

5.2.1 Regression of Fitted Values from Two Models

The two competing models could also be compared by using regression methods. After the MLE of time trend parameters are obtained, the parameters of Beta distribution $(\alpha_t, \beta_t, \delta_t)$ could be calculated for each year. By simulating from these parameters, the sample mean yield as a prediction $\hat{y}_{1t}$ from this model could be calculated for each year. This set of prediction $\hat{y}_{1t}$ could be used as one of the covariates to explain the actual yields.

Similarly, after the MLEs of the detrended yields $\tilde{y}_t \sim Beta(\alpha, \beta, \delta)$ are obtained, the MLEs of the actual yield y_t from this model are $y_t \sim Beta(\alpha, \beta, \delta_t)$ with $\delta_t = \delta * \frac{\hat{y}_t}{\hat{y}_T}$. Thus, a prediction $\hat{y}_{2t}$ could be generated by simulating from the $y_t \sim Beta(\alpha, \beta, \delta_t)$. Then $\hat{y}_{2t}$ could be another covariates to explain the variation of the actual yields. The regression model is shown as:

$$y_t = \gamma_0 + \gamma_1 * \hat{y}_{1t} + \gamma_2 * \hat{y}_{2t}$$

The results show that only the coefficient γ_1 of predicted yields from the time-varying model is significant which implies the time-varying model is better in predicting the actual yield (Table 6).

From the comparison of the time-varying model and the conventional two-stage models, there are significant in-sample evidence of good fit for time-varying distribution and strong out-of-sample evidence in favor of time-varying model as well, which means the time-varying model will lead to more accurate yield distribution estimation in crop insurance practice.

6 Rates Calculation and a Simulation Study

In this section, the impacts of adopting the time-varying rates are evaluated in a simulation study. The premium rates from the proposed time-varying approach are illustrated with simulated data. A cross-rate validation test is conducted to compare the predicted rate error of the time-varying approach with that of the conventional two-stage approach.

By running the simulation from the estimated time-varying model and conventional two-step Beta distribution model respectively, the expected loss and the premium rates are calculated. In this simulation study, 1,000,000 random variables are generated from these time-varying Beta Distributions. For example, when t=1927, 1928, ..., 2006 in model II, 1,000,000 random variables are generated from $Beta(\alpha_t, \beta_t, 0, \delta_t)$. The probability of yield loss and the expected yield loss associated with a contract that guarantees 75 percent of the expected yields of each year are calculated. The corresponding premium rates for this insurance contract are then calculated for every year. The premium rates range from 0.83 percent in 1985 to 0.36 percent in 2006 for an insurance contract guaranteeing 75 percent of the expected yield for the case in which the yields are coming from the time-varying Beta distribution with an exponential time trend structure. While the premium rates calculated from a conventional two-stage Beta distribution model indicate an approximately constant premium around 1.85 percent from 1927 to 2006. In this case, the conventional model tends to over-price for the same level of coverage.

The performance of the time-varying rates was compared to the existing rates by assuming that these are two alternative rating procedures in the crop insurance program. By assuming one of the two models (the time-varying Beta model and the conventional detrended Beta model) is true, for example, the true model is a two-step MLE with detrending parameters b_1, b_2, the cross-rate-validation can be tested. The distribution of the detrended yields $\tilde{y}_t$ is assumed to be true distribution of $Beta(\alpha, \beta, 0, \delta)$, which implies the actual yields with $Beta(\alpha, \beta, 0, \delta_t)$, where $delta_t = \delta * \frac{\hat{y}_t}{\hat{y}_T}$ by the Jocobian transformation of $\tilde{y}_t$ and y_t .This true model implies a true set of premium rates around 1.88 percent from 1927 to 2006. A thousand sets of 80 pseudo-observations are simulated from the "true" yield distribution $Beta(\alpha, \beta, 0, \delta_t), t = 1, \ldots, 80$. Each 80 pseudo-observations are then used to fit the time-varying Beta distribution $Beta(\alpha_t, \beta_t, 0, \delta_t)$ by MLE and 1000 sets of simulated premium rates can be calculated accordingly. The rates calculated from the time-varying model will be compared with the true rate calculated from the "true" conventional model and the difference between them is recorded each time and the mean of these predicted rate errors are calculated yearly.

Similarly, if the true model is time-varying model with true distribution as $Beta(\alpha_t, \beta_t, 0, \delta_t)$, where $t = 1, 2, \ldots, 80$. Each year's Beta can randomly generate 1 realization with a total of 80 realizations through all 80 years. The process is repeated 1000 times and 1000 sets of 80 pseudo-observations are generated. The conventional two-stage model is then used to fit these pseudo-observations. A thousand sets MLE of $Beta(\alpha, \beta, 0, \delta)$ are obtained and the premium rates are calculated accordingly. The predicted rate errors are calculated each year by comparing these predicted rates with the "true" rates derived from the "true" time-varying model.

As shown in Table 8, when the true rate is derived from the conventional model (with an average equal to 0.0188), the mean squared error (MSE) in predicted rates of the time-varying model is 0.0087, which is 9.58% lower than MSE (0.0097) of the conventional model when the alternative (the average premium rate implied by the time-varying model is 0.0058) is true. The result shows that model II predicts more accurate premium rates compared to the expected premium rates from a standard detrended yield data distribution in terms of small rate predicted error.

7 Conclusion

This study has taken a close look at the accuracy of measuring conditional yield risk over time. A more accurate and consistent method is proposed for estimating the distribution of crop yields than previous methods used in the literature. This proposed method of simultaneously estimating the time trend effects and the parameters of the yield distribution overcomes the shortcomings of the typical approach of treating the detrended yield data as though these were "observed" data and thus should improve the accuracy of the time trend and distribution estimates.

Several model selection tools are used to compare the in-sample goodness of fit and out-of-sample prediction power between the conventional two-stage model and the time-varying distribution model. The result shows that the time-varying model is superior to the conventional two-stage model in terms of lowest AIC and BIC and stronger out-of-sample prediction power (the two-year ahead predict errors of time-varying model is 20.04% lower than that of the conventional model and is 20.14% lower for leave-10-out forecast errors). It is important to accurately calculate the actuarially premium rate in the design of crop insurance contract. The premium rate derived from the time-varying model shows a decreasing premium rates (from 0.83% in 1985 to 0.36% in 2006) over years. While the conventional model shows a constant premium rate (1.88%) each year at the same 75% coverage level. As shown in the previous discussion, the conventional model tends to underestimate the yield. As a result, the yield risk tends to be over-estimated and thus a higher premium rate is derived. The result of cross-rate validation test shows that the predicted errors for premium rates of time-varying model are 9.58% lower than the conventional model when the alternative model is true.

This study contributes to the accuracy of assessment of the yield distribution and addresses changes in the parameters of the distribution over time. The conditional yield risks are modeled in a more accurate and systematic way. This will improve the accuracy of the models used in rating crop insurance contracts and are crucial for conducting better risk management to producers from the yield risk.

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Table 1. Equation Estimates: Detrend Crop Yields

Variable	Parameter Estimate	Standard Error	p Value[a]
Corn Yield–Adair County, $R^2 = 0.84$			
Intercept	32.42	5.49	<.0001*
$\tilde{t}$	48.63	31.25	0.12
$\widetilde{t^2}$	131.50	37.40	0.0007*

[a] An "*" indicates statistical significance at the $\alpha = .10$ or smaller level.

Table 2: Maximum Likelihood Estimates for Detrended Yield Data

Model I: 4-parameter Beta—$LLF = -378.69$

parameter	est	s.e.	tstat	95% C.I.
shape1(α)	5.9862	0.2051	29.1802	(5.5841,6.3883)
shape2(β)	2.098	0.2308	9.0912	(1.6457,2.5503)
location(θ)	0.9738	7.8495	0.1241	(-14.4113,16.3589)
scale(δ)	203.4309	1.04	195.6036	(201.3924,205.4693)

Table 3: Maximum Likelihood Estimates for Detrended Yield Data

Model II: 3-parameter Beta— $LLF = -380.67$

parameter	est	s.e.	tstat	95% C.I.
shape1(α)	5.9949	0.1902	31.5176	(5.6221,6.3677)
shape2(β)	2.073	0.2267	9.1441	(1.6287,2.5173)
scale(δ)	204.1263	1.0733	190.1944	(202.0228,206.2299)

Table 4: Maximum-Likelihood Parameter Estimates and Summary Statistics for Time-varying Models II & III

	Linear Trend		Quadratic Trend	
Variables	Estimates	Std. Error	Estimates	Std. Error
b_1	2.5544	0.1939*	2.5517	0.1*
b_2	0.4316	0.7238	0.1569	0.4
b_3	—	—	-0.2888	0.5
b_4	2.9271	0.3324*	2.9529	0.1*
b_5	-2.7095	1.0622*	-1.6313	0.3
b_6	—	—	-2.6135	0.4
b_7	2.1453	5.8693	12.2573	117.7
b_8	-7.4956	13.9888	-15.2701	138.15
b_9	—	—	-13.7221	90.03

Negative LLF(L) $L1 = 328.6832$ $L2 = 326.6221$

LRT Statistics: $-2(L1 - L2) = 4.12$, $\chi^2_{(4df)}$, p-value = 0.3900

Notes: An asterisk denotes statistical significance at the $\alpha = 0.05$ or smaller level

Table 5: Model Comparison Using AIC and BIC (Data: Corn yield in Adair County)

Model	LLF	K	AIC	Δ AIC	BIC	Δ BIC
Detrended Model–3 parameters	-378.6743	3	763.53	94.16	770.48	86.82
Detrended Model–4 parameters	-378.6942	4	765.38	96.01	774.9	91.24
Beta-Time-Varying–3 parameters	-328.6832	6	669.37	0	683.66	0
Beta-Time-Varying-t-square–3 parameters	-326.6221	9	671.24	1.87	692.68	9.02

Note: K is the number of parameters in a model.

Table 6: Two-year-ahead Prediction Error and Ten Out-of-sample Performance

Model	$Error_{2-ahead-pred}$ (1993-2006)	$Error_{10out}$ (1993-2006)
Detrended Model—3 parameters	545.71	241.7
Beta Time-varying model—3 parameters	436.33	193.02

Table 7. OLS for Model Comparison

Variable	Parameter Estimate	p Value^a
Intercept	−0.125	0.97
γ_1	1.068	0.0341*
γ_2	−0.065	0.89

An "*" indicates statistical significance at the $\alpha = .10$ or smaller level.

Table 8: Simulated Premium Rates from 1985 to 2006 (True Model: Conventional Model)

year	True-Rate: Conventional Model	Predicted Error —Time-varying Model
1985	0.018907	0.000051048
1986	0.018896	0.00005287
1987	0.018855	5.44965E-05
1988	0.018837	0.000056555
1989	0.018789	0.000058466
1990	0.018815	0.000060794
1991	0.018942	6.41835E-05
1992	0.018819	0.000065769
1993	0.018897	0.000068978
1994	0.018986	0.000072448
1995	0.018888	7.42865E-05
1996	0.018932	0.000077936
1997	0.018957	8.11015E-05
1998	0.018761	0.000082967
1999	0.018764	0.000086086
2000	0.018839	0.000089985
2001	0.01895	0.000094646
2002	0.018898	0.000097802
2003	0.018898	0.000101405
2005	0.018805	0.000103685
2006	0.018839	0.000108612
	Average=0.0188	MSE=0.0087

Table 9: Simulated Premium Rates from 1985 to 2006 (True Model: Time-varying Model)

year	True-Rate: Time-varying Model	Predicted Error —Conventional Model
1985	0.0083	0.0000562
1986	0.0080	0.0000596
1987	0.0077	0.0000635
1988	0.0074	0.0000673
1989	0.0071	0.0000715
1990	0.0069	0.0000751
1991	0.0066	0.0000782
1992	0.0064	0.0000827
1993	0.0061	0.0000863
1994	0.0059	0.0000895
1995	0.0057	0.0000941
1996	0.0055	0.000098
1997	0.0052	0.0001019
1998	0.0050	0.0001066
1999	0.0048	0.0001107
2000	0.0046	0.0001141
2001	0.0044	0.0001177
2002	0.0042	0.0001217
2003	0.0040	0.0001254
2005	0.0038	0.0001292
2006	0.0036	0.0001334
	Average=0.0058	MSE=0.0097

Table 10: Cross-rate-validation Table (1985 to 2006)

	Conventional True-Rate (0.01887)	Time-varying True-Rate (0.0058)
Conventional Predicted Error	0	0.0096
Time-varying Predicted Error	0.00868	0