

# NON-PRICE EQUILIBRIA FOR NON-MARKETED GOODS<sup>^</sup>

by

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## **I. Introduction**

Empirical studies of market activities draw on an elegant and coherent body of theory that describes household and firm interactions in the market place. Price taking households purchase goods produced by firms that compete to maximize profits under a variety of market power conditions. Theory provides behavioral predictions for households and firms as well as statements about how the aggregation of this behavior results in equilibrium price and quantity outcomes. Models of general equilibrium rely on this link between individual behavior and aggregate outcomes to describe how exogenous changes lead to both direct and indirect effects in price and quantity space. Often it is the indirect, or feedback effects, that are the most interesting in market studies. A variety of empirical and calibration techniques have been developed in economics to study these effects. The modern empirical IO literature focusing on particular industries provides a good example of the former while CGE models of whole sectors of the economy provide good examples of the latter. In both cases the emphasis is on modeling and understanding equilibrium outcomes in price and quantity space.

The story is quite different in studies of non-market goods and activities that are typically employed by environmental economists for purposes of non-market valuation. By definition non-marketed goods are not exchanged in markets, and therefore one cannot speak of equilibrium prices and quantities for the goods per se. Instead the emphasis is usually on understanding preferences in a partial equilibrium framework for a quasi fixed level of a public good. For this purpose an impressive array of structural econometric models has been developed that are capable of predicting individuals' valuations for exogenous changes in the level of the public good. For example, recreation demand modelers use increasingly sophisticated models of quality differentiated demands to understand how recreation site attributes affect behavior and

well-being. Hedonic property value models use ever increasing levels of spatially resolute data to parse out the contribution of a local public good to housing prices. Although the latter models make use of equilibrium concepts to motivate estimation, they rarely are capable of predicting feedback effects from large-scale changes in public good levels. Thus, with few exceptions, it seems reasonable to say that non-market valuation has focused primarily on partial equilibrium analysis of the interactions between behavior and quasi-fixed levels of environmental quality.<sup>1</sup>

This emphasis is probably reasonable in general. Empirical models of behavior that use measurable environmental quality as explanatory variables usually find effects that are of second order importance relative to non-environmental factors. For example, ambient water quality in recreation demand models is usually much less important in explaining water site choice and visitation frequency than travel cost. Likewise, structural characteristics tend to explain much more of the variability in housing prices than does air quality in hedonic property value models. As we describe more fully below, water quality and air quality in these contexts are examples of non-price attributes that we might reasonably suppose to be exogenous to the behavior that we are attaching to them. In contrast, the levels of other types of attributes – such as congestion or angler catch rates in recreation models, or traffic levels in residential location choice models – are at least partially determined by the aggregation of behavior under analysis. We may therefore wonder if there are situations in which general equilibrium feedback effects in endogenous attribute space might be empirically important in non-market valuation. This might be particularly so for a large-scale policy intervention that substantially changes the level and spatial distribution of environmental quality. In this paper we begin to consider the extent to

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<sup>1</sup> There are some notable exceptions that have appeared recently in the literature, including the equilibrium sorting models of Smith *et al.* (2004), Timmins (2003), Timmins and Murdock (2006), and Bayer, Keohane and Timmins (2006). These papers are discussed in further detail below.

which non-price equilibria and feedback effects can be identified and accounted for conceptually and in empirical non-market valuation studies.

To examine this question we proceed as follows. We begin by providing a descriptive overview of how we will think about the concept of ‘non-price equilibria’ in non-market valuation. We suggest working definitions of two general types of non-price equilibriums and offer context and motivation by linking these definitions to specific examples and existing literature inside and outside of environmental economics. We then turn to study a specific type of non-price endogenous attribute: congestion in recreation demand models. We do this using both computable general equilibrium (CGE) and econometric models. We begin by laying out a general modeling structure that will be used in both the econometric and CGE models, which is followed by a description of the Iowa lakes data that motivates our empirical analysis in both modeling frameworks. We then consider an empirical model of recreation demand that explicitly includes site congestion as an explanatory variable, accounts for its econometric endogeneity, and allows computation of both partial and general equilibrium welfare measures. After presenting results from this analysis we turn to the CGE model, which is used to explore more generally situations when partial and general equilibrium welfare measures can be different and under what circumstances it might be important to consider non-price feedback effects in empirical non-market valuation models.

With the three components of this paper we provide three contributions that are in the spirit of the ‘frontiers’ theme of this conference. First, we lay out a research agenda that is motivated by the notion that large scale policy interventions might lead to feedback effects in non-price variables similar to the types that have only just begun to be considered empirically in price space via the new classes of sorting models. Second, we make use of both CGE and

econometric modeling approaches to analyze the feedbacks problem and demonstrate how these quite different tools can shed light on the same problem from different angles. Since the behavior we are interested in is characterized by both intensive and extensive margin decisions both modeling approaches must admit binding non-negativity constraints. Thus a second contribution of this work is the further development of CGE and econometric models that are flexible, tractable, and provide realistic and internally consistent representations of the behavior we are modeling. In the case of the CGE model this involves creative new solution algorithms; in the case of the econometric model this involves the use of contemporary simulation-based econometrics and instrumental variables techniques. The final contribution involves the application to recreation visits to IA lakes and accounting for congestion in the model, an important attribute in recreation demand that is almost always absent in revealed preference demand studies. These three contributions notwithstanding, we stress that this effort is a first, rather than the final, step in this area of inquiry. As such we discuss throughout the simplifications and assumptions we have made and how these motivate topics for further research.

## **II. Conceptual Overview**

To ground our discussion of non-price equilibriums we consider the following behavioral setup. Agents in a closed economy maximize an objective function by choosing levels of activities that are defined by both price and a set of non-price attributes. In the case of consumers the activities are demands for quality-differentiated goods; in the case of firms we can think of them as derived demands for quality-differentiated factor inputs. For the remainder of this discussion we use terminology corresponding to the consumer's problem, although we will also provide examples that correspond to firm's behavior. Households consume the quality

differentiated goods in non-negative quantities and can, at the optimum, be at a corner solution for a subset of the goods in the choice set. The set of non-price attributes that describe the goods in the choice set can be divided into two types: those that are exogenously determined and those that are at least partially determined by the actions of individuals in the model. We refer to the latter as endogenously determined attributes. We are ultimately interested in understanding the extent to which the levels of endogenous attributes might change in response to exogenous or policy shocks, and what the resulting differences are between partial and general equilibrium welfare measures for policy interventions.

This general setup can be better understood by adding a few specific examples. The most obvious case is when the quality differentiated goods are trips to recreation sites, say a set of lakes. The demand for trips depends on individual travel costs as well as attributes of the recreation sites. Attributes such as the presence of boat ramps, picnic facilities, and perhaps ambient water quality are exogenous to the decision process. In contrast, congestion at the recreation sites is determined by the aggregate visitation decisions of the people in the economy and is therefore an endogenous attribute. Similarly, angling catch rates for sport fish species at the lakes are determined not only by existing bio-physical conditions, but also by the spatial and temporal distribution of anglers' fishing effort. Policy interventions such as water quality improvements, facility improvements, or fish stocking programs might have direct welfare effects as well as indirect effects that play through via the re-equilibrating of congestion and catch rate attributes that change due to people's changed visitation patterns.

A second example is the choice of residential location, which conveys a bundle of market and non-market services. Exogenous attributes in the bundle include characteristics of the structure and distance to natural features such as lakes. Endogenous attributes might include

traffic congestion and the resulting local air quality impacts, privately held open space, and publicly held open space created by local governance. This latter attribute is related to other endogenous attributes that have more of a public finance or urban economic flavor, such as local school quality and the racial makeup of neighborhoods. We return to this below as examples of topics outside environmental economics in which empirical work on non-price equilibria has advanced.

A final example comes from commercial fisheries. Vessel operators choose the timing and location of harvest effort that gives rise to an aggregate distribution of effort in much the same way that congestion is determined in a recreation model. This effort, in combination with the biological system, gives rise to equilibrium populations for the targeted species as well as equilibrium levels of spatially and temporally distributed catch effort.

Based on these examples, we define two general types of non-price equilibria that can occur in environmental economics applications. The first case we refer to generically as a *simple sorting equilibrium*. In this case levels of endogenous non-price attributes are determined only by interactions between agents. Among the examples mentioned this class includes congestion in recreation applications and more generally social interaction outcomes such as racial mixing and peer effects in schools. In these cases the equilibrium outcomes are determined only by the interactions/feedbacks among agents. This is in contrast to the second type of non-price equilibrium that we define which we label a *complex sorting equilibrium*. This refers to situations in which the agents interact with a quasi-supplier of environmental conditions, which will often be the natural environment, to determine the equilibrium. Among the examples we have mentioned recreation fishing catch rates fall into this class. Here the natural environment (via population dynamics and available habitat) provides the stock of fish while anglers'

aggregate distribution of trips and catch effort provides the level of stock exploitation. The interaction results in equilibrium catch rate levels and fish populations. Educational outcomes in public finance applications are similarly complex sources of sorting equilibrium. Here the sorting behavior of households into school districts determines peer effects, while district funding levels (also partially determined by sorting behavior if there is local control of schools) determine teacher and facility quality. Together these factors determine educational outcomes.

To formalize these ideas consider the following model of consumer behavior related to the choice of quality differentiated goods. There are  $N$  consumers in the economy, each of whom maximizes utility by choosing the levels of a  $J$ -dimension vector of quality differentiated goods denoted by  $z_i$  and spending on other goods  $x_i$ . The problem is given analytically by

$$\max_{z_i, x_i} U_i = U_i(z_i, x_i; Q) \quad s.t. \quad p'_i z_i + x_i \leq y_i, \quad z_{ij} \geq 0, \quad j = 1, \dots, J, \quad (1)$$

where  $U_i$  is the utility index for person  $i$ ,  $Q$  is an  $M \times J$  matrix of non-price attributes where  $M$  is the number of attributes,  $p_i$  is the vector of prices for the quality differentiated goods, and  $y_i$  denotes the person's income. Some or all of the quality attributes in the model may be endogenously determined by the aggregation of consumers' behavior and aspects of the natural environment. Thus we define an *attribute transmission function* by

$$q_{mj} = q_{mj}(z_1, \dots, z_N; E), \quad m = 1, \dots, M, \quad j = 1, \dots, J, \quad (2)$$

where  $q_{mj}$  is an individual element of the quality matrix  $Q$ ,  $z_1, \dots, z_N$  denotes the aggregate behavior by households in the economy, and  $E$  stands for the natural environment or a quasi-supplier. The transmission function shown in (2) is general in that it describes an endogenous attribute arising from a complex sorting equilibrium, but it also nests as special cases the two additional types of attributes we have defined:

- $q_{mj} = q_{mj}(E)$  for exogenous attributes

- $q_{mj}=q_{mj}(z_1, \dots, z_N)$  for endogenous attributes arising from a simple sorting equilibrium.

In making their choices we assume individuals take levels of the attributes as given, so the first-order conditions for person  $i$  are:

$$\frac{\partial U_i(z_i, x_i; Q) / \partial z_{ij}}{\partial U_i(z_i, x_i; Q) / \partial x_i} \leq p_{ij}, \quad z_{ij} \left[ \frac{\partial U_i(z_i, x_i; Q) / \partial z_{ij}}{\partial U_i(z_i, x_i; Q) / \partial x_i} - p_{ij} \right] = 0, \quad z_{ij} \geq 0, \quad j = 1, \dots, J. \quad (3)$$

Equilibrium in the economy is characterized by the simultaneous solution to a system of equations/inequalities and their associated complementary slackness relationships that correspond to:

- The first-order conditions described by (3) for all individuals  $i = 1, \dots, N$
- The attribute transmission functions in (2) for  $j=1, \dots, J$  and  $m \in \tilde{M}$ , where  $\tilde{M} \in M$  is the set of endogenous attributes.

In defining non-price equilibria we have thus far tried to be fairly general, but it is clear that this can only be taken so far. Unlike price and quantity equilibrium concepts for homogenous goods, for which theory provides quite general results, non-price equilibria for quality differentiated goods are by definition context specific. The challenge for applied welfare analysis is to characterize conceptually and empirically the particulars of the equilibrium of interest. Simple sorting equilibria seem easier to deal with than their complex counterparts in that for the former the analyst need only specify the mechanism through which agents interact, while for the latter agent interactions, a production function, and the relationship between the two must be spelled out. Nonetheless there are relatively few examples of applied work dealing with either type of non-price equilibrium. Thus after providing a brief review of related literature we consider the specific example of congestion in recreation demands models.

### III. Literature Review

Concerns regarding equilibrium sorting on the basis of endogenous quality attributes are not new to the literature, though they have typically taken a back seat to sorting driven by market price movements. Schelling (1978) provides one of the earlier discussions of non-price sorting, illustrating qualitatively how non-market adjustments might lead to surprising equilibrium outcomes, most notably, perhaps, in the context of racial segregation. More recently, there have been efforts to estimate and calibrate CGE models driven by exogenous and endogenous quality attributes. Ferreyra (2006), for example, examines the general equilibrium impacts of school voucher programs, with households sorting on the basis of both location and school quality. Private and public school quality are determined in equilibrium by the composition of households within a district. The parameters of the model are estimated using school district, rather than household, level data on income, rental values, etc. However, household level preference heterogeneity is allowed for in the general equilibrium simulations by assuming a distribution of individual agents in the population with varying preferences for school quality and religious affiliation.

Within the environmental literature, there are three broad strands of research related to the current paper. The first strand is the locational equilibrium sorting literature exemplified by work of Smith, *et al.* (2004), which in turn draws on the work of Epple and Sieg (1999) and Epple, Romer and Sieg (2001). The authors, in this case, examine the equilibrium effects from an *exogenous* change in air quality stemming from reduced ozone levels. Sorting then occurs on the basis of preferences for housing, education and air quality, estimated using all individual home sales records for the LA Air Basin between 1989 and 1991. The authors find that accounting for the GE responses to the air quality changes can substantially alter the implied benefits from ozone reductions in the region, particularly for specific counties within the basin.

The second strand of literature related to the current paper includes the locational sorting models developed by Bayer and Timmins (2005,2006) and applied to environmental issues by Bayer, Keohane, and Timmins (2006), Timmins and Murdock (2006), and O'Hara (2006). The distinguishing feature of the Bayer and Timmins approach is that they allow the quality characteristic of interest to be determined endogenously. For example, in Timmins and Murdock (2006), the authors allow recreational site quality to be determined in part by the level of congestion at the site. A two-stage estimation process is used to estimate a random utility maximization (RUM) model of site selection as a function of travel costs and congestion, while controlling for the endogeneity of congestion. The authors find that ignoring congestion effects can substantially underestimate the value of a lake. While we employ a similar two-stage approach for estimating the impact of congestion on site preferences, the current work differs from that of Timmins and Murdoch (2006) in that we model not only site selection, but also the intensity of usage (i.e., numbers of trips) for each site and employ a Kuhn-Tucker dual modeling structure rather than the RUM framework.

Finally, Massey, Newbold and Gentner (2006) examine the impact of water quality improvements on recreational fishing in Maryland's costal bays. The unique feature of their analysis is that, while they consider only exogenous water quality changes, these changes impact recreational fishing indirectly through a bio-economic model of the coastal fishery. Their approach of explicitly modeling the dynamic evolution of fish stock in response to water quality changes provides a natural starting point in considering the sort of complex sorting discussed in the previous section in which endogenous attributes such as visitation rates or other measures of congestion interact with biological conditions at the site to determine site attributes of interest to recreationists, such as fish catch rates.

#### **IV. Modeling Framework**

In order to assess the feedback effects we are interested in, the non-price equilibrium outcomes need to be linked in a consistent manner to estimable models of individual behavior. Specifically, the models must be able to capture consumer response to changing price and quality conditions and allow for those responses to occur at both the intensive and extensive margins. This is particularly important when evaluating major policy shifts that will induce some individuals to enter or leave the market entirely (i.e., when corner solutions emerge). What is required is a model that readily admits the concept of a virtual price. By virtual price we mean a summary measure that consistently and succinctly captures all constraints on behavior, both price and non-price, which in turn ultimately determines people's choices. The KT model (Phaneuf *et al.* 2000; von Hafen *et al.* 2004) and its dual counterpart (Lee and Pitt 1986; Phaneuf 1999) are uniquely positioned for this purpose. The latter is particularly attractive in that it involves a direct parameterization of individuals' virtual prices. Estimation and welfare calculations (and examining equilibrium concepts in the model) involve comparisons among virtual prices. The flexibility in characterizing behavior afforded by the dual model is also an attractive feature of the approach.

In this section of the paper we provide a general overview of the dual modeling framework and a discussion of the particular functional form that will be used in our application and CGE analysis. To estimate the model we draw upon Bayesian tools of data augmentation/Gibbs sampling and a two-step procedure for handling the underlying endogeneity of quality attributes (such as congestion and fish stock) that are central to our investigation of equilibrium responses. We delay discussion of these details until section VI, which follows our description of the application in section V.

### A. The Dual Model

The dual model begins with the specification of the individual's underlying indirect utility function. Dropping the individual subscripts for the moment let  $H(p, y; Q, \theta, \varepsilon)$  denote the solution to a utility maximization problem defined by:

$$H(p, y; Q, \theta, \varepsilon) = \underset{z}{\text{Max}} \{U(z; Q, \theta, \varepsilon) \mid p'z + x = y\}, \quad (4)$$

where as above  $(z, x)$  denotes the private goods to be chosen by the individual,  $p$  is the vector of corresponding prices,  $y$  denotes income, and  $Q$  is a vector or matrix of public goods that the individual agent takes as given. In our application considering the demand for recreation the private goods include recreation trips to the available sites and all other spending,  $p$  reflects the costs of traveling to those sites, and  $Q$  includes the quality attributes of the sites. These attributes, while taken as given by the individual, include factors (such as congestion and fish stock) that are determined in equilibrium by the decisions made in the market as a whole.<sup>2</sup> The direct utility function  $U(z; q, \theta, \varepsilon)$  depends upon parameters  $\theta$  and attributes of the individual  $\varepsilon$  that are unobserved by the analyst.

It is important to note that the indirect utility function in equation (4) is derived without imposing non-negativity constraints on demand. Applying Roy's Identity to equation (4) thus yields *notional* (or latent) demand equations

$$z_j^* = \frac{\frac{\partial H(p, y; Q, \theta, \varepsilon)}{\partial p_j}}{\sum_{k=1}^J p_k \frac{\partial H(p, y; Q, \theta, \varepsilon)}{\partial p_k}}, \quad j = 1, \dots, J, \quad (5)$$

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<sup>2</sup> Furthermore, some of the site attributes will not be observed by the analyst. For now we do not distinguish between observed and unobserved quality attributes, though this distinction will become important at the estimation stage.

where  $z_j^* = z_j^*(p, y; Q, \theta, \varepsilon)$  will be negative for goods the individual does not wish to consume and positive for those goods she does consume. Observed consumption levels are then derived through the use of *virtual* prices, which rationalize the observed corner solutions. For example, suppose that the first  $r$  goods are not consumed. Let  $p_N = (p_1, \dots, p_r)'$  denote the prices for the non-consumed (i.e., corner solution) goods and  $p_C = (p_{1+r}, \dots, p_J)'$  denote the prices for the consumed goods (i.e., those with positive consumption). The virtual prices for the non-consumed goods are implicitly defined by

$$0 = \frac{\partial H(\pi_1(p_C; Q, \theta, \varepsilon), \dots, \pi_r(p_C; Q, \theta, \varepsilon), p_C, y; Q, \theta, \varepsilon)}{\partial p_j}, \quad j = 1, \dots, r. \quad (6)$$

The observed demands for all the commodities become

$$z_j = \frac{\frac{\partial H(p^*, y; Q, \theta, \varepsilon)}{\partial p_j}}{\sum_{k=1}^J p_k^* \frac{\partial H(p^*, y; Q, \theta, \varepsilon)}{\partial p_k}} = z_j(p^*, y; Q, \theta, \varepsilon), \quad (7)$$

where  $p^* = (\pi_1, \dots, \pi_r, p_{1+r}, \dots, p_J)'$ . For the non-consumed goods, we have that

$$z_j(p^*, y; Q, \theta, \varepsilon) = 0, \quad j = 1, \dots, r \quad (8)$$

by construction and

$$\pi_j(p^*, y; Q, \theta, \varepsilon) \leq p_j, \quad j = 1, \dots, r. \quad (9)$$

For the consumed goods, in contrast, we have:

$$z_j(p^*, y; Q, \theta, \varepsilon) = z_j, \quad j = r+1, \dots, J, \quad (10)$$

and

$$\pi_j(p^*, y; Q, \theta, \varepsilon) = p_j, \quad j = r+1, \dots, J. \quad (11)$$

The system of equations in (8) through (11) provides the link between the observed data on usage and the implied restrictions on the underlying error distributions, which can be in turn used for the purposes of estimation. In addition these equations allow us to express the actual indirect utility function (the solution to the utility maximization problem with non-negativity constraints enforced) as

$$V(p, y; Q, \theta, \varepsilon) = \max_{\omega \in \Omega} \{H(p^\omega, y; Q, \theta, \varepsilon)\}, \quad (12)$$

where  $\Omega$  denotes the set of all possible *demand regimes* (combinations of corner and interior solutions among the  $J$  sites) and  $p^\omega$  denotes the particular combination of virtual and actual prices associated with demand regime  $\omega$ .

Equation (12) demonstrates explicitly that the dual model is an endogenous regime-switching approach. The solution to the consumer's problem consists of the set of goods (recreation sites in our context) that are chosen as well as the quantity of each good consumed. The comparison of true and virtual prices as shown in equation (9) distinguishes the chosen goods from the non-chosen goods and thus can be used to gauge movements into and out of the market for particular commodities. In addition the virtual prices are functionally dependent on both the price and non-price attributes of the goods. Thus it is appropriate to view the virtual prices as quality-adjusted endogenous reservation prices, which can change in response to either price or non-price attribute changes. In the recreation context we can therefore measure changes in visitation patterns to particular sites by examining how virtual prices change in response to both exogenous and endogenous attribute changes.

### B. *Model Specification*

The dual model specification can be obtained by choosing a functional form for the underlying indirect utility function  $H(p, y; Q, \theta, \varepsilon)$  and deriving from it the corresponding notional

demand and virtual price equations (see for example Pitt and Millimet, 2003). Alternatively, as in Wang (2003), one can begin with a system of notional demands for the goods of interest (i.e., an incomplete demand system) and integrate back to obtain the underlying quasi-indirect utility function (Hausman, 1981). The advantage of the latter approach is that tractable notional demand equations can be specified, making the computation of virtual prices straightforward. The disadvantage here, of course, is that the resulting demand system is incomplete and does not capture substitution possibilities to goods outside of the choice set. In addition the restrictions needed to ensure integrability tend to require a choice between substitution and income effects. While we employ the incomplete demand system approach in our empirical analysis, further research into models that start with the underlying indirect utility function (e.g., using a Hicksian composite for all other goods) seems worth pursuing.

Our empirical specification begins with the following system of Marshallian notional demand equations

$$z_{ij}^* = \alpha_j + \sum_{k=1}^J \beta_{jk} p_{ik} + \gamma_j y_i + \varepsilon_{ij}, \quad j = 1, \dots, J, \quad (13)$$

where the subscript  $i$  denotes people  $i=1, \dots, N$  and  $\alpha_j = \alpha_j(q_j, \xi_j)$  is a function of both observable site attributes for site  $j$  (the vector  $q_j$  – the component of  $Q$  specific to site  $j$ ) and unobservable factors which we denote  $\xi_j$ . We assume that these factors have a linear form given by

$$\alpha_j = \gamma' q_j + \xi_j, \quad j = 1, \dots, J. \quad (14)$$

The remaining notation is as follows:  $p_{ik}$  is the price of site  $k$  for individual  $i$  and  $y_i$  denotes individual  $i$ 's income level. We impose a series of restrictions on the parameters of (13) to make the system of equations weakly integrable (See LaFrance 1985, 1986). Specifically, we assume that  $\beta_{jk} = \beta_{kj} \forall j, k$  and  $\gamma_j = 0 \forall j$ . The resulting notional demand system becomes:

$$z_{ij}^* = \alpha_j + \sum_{k=1}^J \beta_{jk} p_{ik} + \varepsilon_{ij}, \quad j = 1, \dots, J. \quad (15)$$

The corresponding (notional) quasi-indirect utility function for (15) is given by:<sup>3</sup>

$$\tilde{H}(p_i, y_i; Q, \theta, \varepsilon_i) = y_i - \sum_{j=1}^J [\alpha_j + \varepsilon_{ij}] p_{ij} + \sum_{j=1}^J \sum_{k=1}^J \beta_{jk} p_{ij} p_{ik}. \quad (16)$$

As indicated above the notional demand equations can be used to define virtual prices for the non-consumed goods that rationalize the observed corner solutions. For illustration, suppose again that the first  $r$  goods are not consumed. The virtual prices  $\pi_{ij} = \pi_{ij}(p_{iC}; Q, \theta, \varepsilon_i)$  for these commodities are then implicitly defined by the system of equations:

$$0 = \alpha_j + \sum_{k=1}^r \beta_{jk} \pi_{ik} + \sum_{k=r+1}^J \beta_{jk} p_{ik} + \varepsilon_{ij}, \quad j = 1, \dots, r, \quad (17)$$

or in matrix notation by

$$\mathbf{0}_r = \alpha_N + \beta_{NN} \pi_{iN} + \beta_{NC} p_{iC} + \varepsilon_{iN}, \quad (18)$$

where  $\mathbf{0}_r$  is an  $r \times 1$  vector of zeros,  $\alpha_N = (\alpha_1, \dots, \alpha_r)'$ ,  $\pi_{iN} = (\pi_{i1}, \dots, \pi_{ir})'$ ,  $p_{iC} = (p_{i,r+1}, \dots, p_{iJ})'$ ,

$\varepsilon_{iN} = (\varepsilon_{i1}, \dots, \varepsilon_{ir})'$ ,

$$\beta_{NN} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1r} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{r1} & \beta_{r2} & \cdots & \beta_{rr} \end{bmatrix}, \quad (19)$$

$$\beta_{NC} = \begin{bmatrix} \beta_{1,r+1} & \beta_{1,r+2} & \cdots & \beta_{1J} \\ \beta_{2,r+1} & \beta_{2,r+2} & \cdots & \beta_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{r,r+1} & \beta_{r,r+2} & \cdots & \beta_{rJ} \end{bmatrix}, \quad (20)$$

<sup>3</sup> Note that the quasi-indirect utility function depends upon income and the prices of the goods in the incomplete demand system, but not the prices of goods outside of the system. This captures the indirect utility associated with an indirectly weakly separable branch of overall utility (See, e.g., Herriges, 1983; Caves, Christensen and Herriges, 1987).

and

$$\beta_{CC} = \begin{bmatrix} \beta_{r+1,r+1} & \beta_{r+1,r+2} & \cdots & \beta_{1J} \\ \beta_{r+2,r+1} & \beta_{r+2,r+2} & \cdots & \beta_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{J,r+1} & \beta_{J,r+2} & \cdots & \beta_{JJ} \end{bmatrix}. \quad (21)$$

Solving for the virtual prices, we find that

$$\pi_{iN} = -\beta_{NN}^{-1} [\alpha_N + \beta_{NC} p_{iC} + \varepsilon_{iN}] \quad (22)$$

and note that the virtual prices for the non-consumed goods depend on the quality attributes of these goods. The observed demand equations for the consumed goods are derived by substituting (22) into (15), yielding:

$$\begin{aligned} z_{iC} &= \alpha_C + \beta_{CN} \pi_{iN} + \beta_{CC} p_{iC} + \varepsilon_{iC} \\ &= \alpha_C + -\beta_{CN} \beta_{NN}^{-1} [\alpha_N + \beta_{NC} p_{iC} + \varepsilon_{iN}] + \beta_{CC} p_{iC} + \varepsilon_{iC} \\ &= (\alpha_C - \beta_{CN} \beta_{NN}^{-1} \alpha_N) + (\beta_{CC} - \beta_{CN} \beta_{NN}^{-1} \beta_{NC}) p_{iC} + (\varepsilon_{iC} - \beta_{CN} \beta_{NN}^{-1} \varepsilon_{iN}) \\ &= \tilde{\alpha}_C + \tilde{\beta}_{CC} p_{iC} + \tilde{\varepsilon}_{iC}, \end{aligned} \quad (23)$$

where  $\tilde{\alpha}_C \equiv \alpha_C - \beta_{CN} \beta_{NN}^{-1} \alpha_N$ ,  $\tilde{\beta}_{CC} = \beta_{CC} - \beta_{CN} \beta_{NN}^{-1} \beta_{NC}$ , and  $\tilde{\varepsilon}_{iC} = \varepsilon_{iC} - \beta_{CN} \beta_{NN}^{-1} \varepsilon_{iN}$ . Notice that the observed demands depend directly only on the prices of those goods consumed, but that they depend upon the quality attributes for all the goods since all the  $\alpha_j$ 's enter into equation (23) and they each depend in turn upon the corresponding quality attributes.

Finally, in our empirical analysis below, we impose two additional simplifying restrictions on the model in equation (15). First, we assume that all of the own and cross-price coefficients are the same across sites; i.e.,  $\beta_{jj} = \beta_1 \forall j$  and  $\beta_{jk} = \beta_2 \forall j \neq k$ . Second, we assume that the idiosyncratic individual heterogeneity captured by  $\varepsilon_i = (\varepsilon_{i1}, \dots, \varepsilon_{iJ})'$  is *iid*  $N(0, \Sigma)$ , where  $\Sigma = \text{diag}(\sigma_1^2, \dots, \sigma_J^2)$ . Neither of these restrictions is necessary from a conceptual or

computational perspective, but they substantially reduce the number of parameters that must be estimated in the empirical analysis.

## **V. Application**

Our empirical analysis focuses on modeling the demand for lake recreation in Iowa, drawing on data from the first year of the Iowa Lakes Project. The Iowa Lakes Project is a four-year panel data study, analyzing the visitation patterns to 129 Iowa Lakes by 8000 randomly selected households, providing a rich source of variation in usage patterns. Iowa is particularly well suited for our research for several reasons. First, Iowa's lakes are characterized by a wide range of water quality conditions, including both some of the cleanest and some the dirtiest lakes in the world. Second, detailed information is available on the environmental conditions in each lake, including fifteen physical and chemical measures (e.g., Secchi transparency, total nitrogen, etc.) obtained three times each year during the course of the Iowa Lakes Project. Third, the State of Iowa is currently considering major lake water remediation efforts. These include multi-million dollar projects to improve individual target lakes as well as the Governor's stated objective to remove all of Iowa's lakes from the US EPA's impaired water quality list by 2010. These changes provide natural, and policy relevant, sets of scenarios to consider in investigating the general equilibrium effects of regulatory interventions.

The first year Iowa Lakes Project survey was administered by mail to a randomly selected sample of 8000 households beginning in November of 2002. A total of 4423 surveys were completed, for a 62% response rate once non-deliverables surveys are accounted for.<sup>4</sup> While the survey contained a number of sections soliciting information regarding the socio-demographic characteristics of each respondent and their attitudes toward potential water quality

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<sup>4</sup> Details of the survey design and implementation can be found in Azevedo, *et al.* (2003). A copy of the survey instrument is available online at [http://www.card.iastate.edu/environment/items/IowaLakesSurvey\\_02.pdf](http://www.card.iastate.edu/environment/items/IowaLakesSurvey_02.pdf).

changes in the state, the key section for the current analysis obtained information regarding the respondents' visits to each of 129 lakes during 2002. On average, approximately 63% of Iowa households were found to visit at least one of these lakes in 2002, with the average number of day trips per household per year being 8.1. There are, of course, a large number of corner solutions in this dataset, with 37% of the households visiting none of the lake sites and most of the households who choose to visit the lakes visiting only a small subset of the available sites. Fewer than 10% of those surveyed visited more than five distinct sites during the course of a year. For the purposes of the econometric analysis below, a sub-sample of the 2002 usage data was used. Specifically, we had available 1286 observations, randomly selected from the full sample. These records were further narrowed to the 749 users in the sub-sample (i.e., those households taking at least one recreational trip in 2002).

The Iowa Lakes survey data was supplemented with two data sources. First, for each individual in the sample, travel costs from their home to each of 129 lakes were calculated. The transportation software package PCMiller was used to establish both the distance and travel time to each lake. Travel costs were then computed using a travel cost of \$0.28/mile and valuing the travel time at one-third the individual's wage rate. The average travel cost over all site/individual combinations was \$135. Second, water quality measures for each of the lakes were provided by the Iowa State Limnological Laboratory, which had measured water quality at each of the 129 lakes three times in 2002. The average Secchi transparency and Chlorophyll levels are used in the current analysis. Secchi transparency measures water clarity and ranges from 0.09 to 5.67 meters across the 129 lakes in our sample. Chlorophyll is an indicator of phytoplankton plant biomass which leads to greenness in the water, and ranges from 2 to 183  $\mu\text{g/liter}$  among the Iowa lakes.

## VI. Estimation Algorithm

The estimation of the parameters in our model of recreation demand can be divided into two stages. In the first stage, we estimate basic parameters in the demand system characterized by equations (8) through (11). Specifically, we obtain estimates of the  $2J+2$  parameters  $\theta=(\alpha_1, \dots, \alpha_J, \beta_1, \beta_2, \sigma_1, \dots, \sigma_J)$  using a Bayesian computational approach relying on Gibbs sampling and data augmentation.<sup>5</sup> Note that site specific intercept terms capture all of the site specific attributes, including the endogenous factors of interest and unobserved site characteristics. The purpose of the second stage then is to estimate the functional relationship between these intercepts and the observed quality attributes.

### A. First Stage Estimation

Data augmentation techniques pioneered by Albert and Chib (1993) in the context of discrete choice models provide a powerful tool for handling latent variables, simulating these missing components of the data and, in doing so, making the analysis more tractable. In the current context, the latent variables are the notional demands. Together with Gibbs sampling techniques we can use data augmentation to readily simulate values from the posterior distribution of interest.<sup>6</sup>

Formally, the posterior distribution is characterized by

$$p(z^*, \theta | z) \propto p(z | z^*, \theta) p(z^* | \theta) p(\theta). \quad (24)$$

Note that the data augmentation procedure treats the unknown latent factors essentially as additional parameters, characterized in terms of a prior distribution and for which a posterior distribution is generated. The priors for  $\theta$  are assumed to take the following forms:

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<sup>5</sup> More precisely, we obtain posterior distributions on these basic parameters given our data and prior distributions for the same parameters. However, the approach employed here can also be interpreted from a frequentist perspective since, based on the Bernstein-von Mises Theorem, the posterior mean converges to the maximum of likelihood, yielding MLE estimates (See, e.g., Train, 2003, ch. 12).

<sup>6</sup> A similar approach was employed by Wang (2003) and Pitt and Millimet (2003).

$$\psi \equiv \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \sim N(\bar{\psi}, \tau I_m), \quad (25)$$

where  $\tau$  is a large constant and  $I_m$  is an  $m \times m$  identity matrix with  $m=J+2$ , and the  $\sigma_j^2$  are independent inverted gamma variates with  $\sigma_j^2 \sim IG(1, \underline{s}_j)$ . While the joint posterior distribution in (24) is complex, the corresponding conditional posterior distributions for  $z^*$ ,  $\psi$ , and  $\sigma_j^2$  each have convenient functional forms. Thus, an iterative Gibbs sampling routine can be used to simulate draws from the posterior distribution. The following steps are involved in a simulation process drawing  $M$  values from the posterior distribution:

Step 1: Set starting values

Initial values  $\psi(0)$  and  $\sigma_j^2(0)$  for the simulation are established. For example, one might obtain starting values in the current context by running a simple tobit model for each site using observed trips data, yielding site specific intercepts and variance terms (i.e.,  $\alpha_j$  and  $\sigma_j^2$ ) and, by averaging across sites, starting values for the own- and cross-price parameters.

Step 2: Drawing posterior notional demands from  $p(z^* | z, \psi, \Sigma)$

The notional demands, conditional on observed demands and the parameters of the model, are drawn from a truncated normal distribution for non-consumed goods and from a normal distribution for the consumed goods. Specifically, from equation (23) we have that

$$\tilde{\varepsilon}_{ij}(m) = z_{ij} - \tilde{\alpha}_j(m-1) - \tilde{\beta}_{jC}(m-1)p_{iC}, \quad j = r+1, \dots, J, \quad (26)$$

where the parenthetical arguments denote the step in the iteration process ( $m=1, \dots, M$ ). Since

$\tilde{\varepsilon}_{iC} = \varepsilon_{iC} - \beta_{CN} \beta_{NN}^{-1} \varepsilon_{iN}$ , then  $(\varepsilon'_{iN}, \tilde{\varepsilon}'_{iC})' \sim N(0, \Omega)$  where

$$\begin{aligned}
\Omega = \text{Cov} & \begin{bmatrix} \varepsilon_{i1}\varepsilon_{i1} & \cdots & \varepsilon_{i1}\varepsilon_{ir} & \varepsilon_{i1}\tilde{\varepsilon}_{i,r+1} & \cdots & \varepsilon_{i1}\tilde{\varepsilon}_{ij} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{i1}\varepsilon_{ir} & \cdots & \varepsilon_{ir}\varepsilon_{ir} & \varepsilon_{ir}\tilde{\varepsilon}_{i,r+1} & \cdots & \varepsilon_{ir}\tilde{\varepsilon}_{ij} \\ \varepsilon_{i1}\tilde{\varepsilon}_{i,r+1} & \cdots & \varepsilon_{ir}\tilde{\varepsilon}_{i,r+1} & \tilde{\varepsilon}_{i,r+1}\tilde{\varepsilon}_{i,r+1} & \cdots & \tilde{\varepsilon}_{ij}\tilde{\varepsilon}_{i,r+1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{i1}\tilde{\varepsilon}_{ij} & \cdots & \varepsilon_{ir}\tilde{\varepsilon}_{ij} & \tilde{\varepsilon}_{i,r+1}\tilde{\varepsilon}_{ij} & \cdots & \tilde{\varepsilon}_{ij}\tilde{\varepsilon}_{ij} \end{bmatrix} \\
& = \begin{bmatrix} \Sigma_{NN} & \Sigma_{NC} - \Sigma_{NN}\beta_{NN}^{-1}\beta_{NC} \\ \Sigma_{CN} - \beta_{CN}\beta_{NN}^{-1}\Sigma_{NN} & \Sigma_{CC} + \beta_{CN}\beta_{NN}^{-1}\Sigma_{NN}\beta_{NN}^{-1}\beta_{NC} - \beta_{CN}\beta_{NN}^{-1}\Sigma_{NC} - \Sigma_{CN}\beta_{NN}^{-1}\beta_{NC} \end{bmatrix}.
\end{aligned} \tag{27}$$

In the current application, this expression reduces considerably, since  $\Sigma_{NC} = \mathbf{0}_{N \times C}$ , so that:

$$\Omega = \begin{bmatrix} \Sigma_{NN} & -\Sigma_{NN}\beta_{NN}^{-1}\beta_{NC} \\ -\beta_{CN}\beta_{NN}^{-1}\Sigma_{NN} & \Sigma_{CC} + \beta_{CN}\beta_{NN}^{-1}\Sigma_{NN}\beta_{NN}^{-1}\beta_{NC} \end{bmatrix}. \tag{28}$$

The notional demands for the non-consumed goods correspond to  $z_{ij}^* < 0$ , or equivalently:

$$\varepsilon_{ij} < - \left[ \alpha_j + \sum_{k=1}^J \beta_{jk} p_{ik} \right] \equiv \delta_{ij}, \quad j = 1, \dots, r. \tag{29}$$

Using the expression in (29), we sequentially obtain  $\varepsilon_{ij}(m) | \varepsilon_{i,-j}(m)$ , where

$\varepsilon_{i,-j}(m) = (\varepsilon_{i,1}(m), \dots, \varepsilon_{i,j-1}(m), \varepsilon_{i,j+1}(m-1), \dots, \varepsilon_{i,r}(m), \tilde{\varepsilon}_{iC}(m))$  denotes all of the errors for

individual  $i$  except  $\varepsilon_{ij}(m)$ , noting that

$$\varepsilon_{ij} | \varepsilon_{i,-j} \sim TN_{(-\infty, \delta_{ij})}(\mu_{ij|-j}, \omega_{j|-j}), \quad j = 1, \dots, r, \tag{30}$$

where

$$\mu_{j|-j} = \Omega_{j,-j} \Omega_{-j,-j}^{-1} \varepsilon_{i,-j}, \quad j = 1, \dots, r, \tag{31}$$

and

$$\omega_{j|-j} = \omega_{jj} - \Omega_{j,-j} \Omega_{-j,-j}^{-1} \Omega_{-j,j}, \quad j = 1, \dots, r. \tag{32}$$

Draws of  $\varepsilon_{ij}(m)$  for  $j=1, \dots, r$  are obtained using (30) and the inversion method. Values of  $\varepsilon_{ij}(m)$

for  $j=r+1, \dots, J$  are then obtained using

$$\varepsilon_{ij}(m) = \tilde{\varepsilon}_{ij}(m) + \beta_{CN} \beta_{NN}^{-1} \varepsilon_{iN}(m), \quad j = r+1, \dots, J. \quad (33)$$

Finally, the notional demands are formed using

$$z_{ij}^*(m) = \alpha_j(m-1) + \sum_{k=1}^J \beta_{jk}(m-1) p_{ik} + \varepsilon_{ij}(m), \quad j = 1, \dots, J. \quad (34)$$

Step 3: Drawing posterior covariance terms from  $p(\Sigma | z^*, z, \psi)$

The variance terms  $\sigma_j^2$ 's are updated using the conditional posterior distributions

$\sigma_j^2 | z_j^*, \psi \sim IG(1+N, \bar{s}_j)$ , where  $\bar{s}_j = (\underline{s}_j + N s_j^2) / (1+N)$  and  $s_j^2$  denotes the sampling variance

from the ordinary least squares estimation of

$$z_{ij}^*(m) = \alpha_j(m) + \sum_{k=1}^J \beta_{jk}(m) p_{ik}, \quad j = 1, \dots, J. \quad (35)$$

The updated value of  $\sigma_i^2(m)$  is drawn from  $IG(1+N, \bar{s}_j(m))$ .

Step 4: Drawing posterior parameters from  $p(\psi | z^*, z, \Sigma)$

Conditional on the notional demands and the variance-covariance structure, the posterior

distribution of the parameters  $\psi = (\alpha', \beta_1, \beta_2)$  is normally distributed. Specifically,

$$\psi \sim N(\hat{\psi}, \hat{\Sigma}_\psi), \quad (36)$$

where  $\hat{\psi}$  denotes the SUR estimates of  $\psi$  and  $\hat{\Sigma}_\psi$  denotes the corresponding covariance matrix

of the estimated parameters. The parameter vector  $\psi(m)$  is then obtained by drawing from

$$N(\hat{\psi}(m), \hat{\Sigma}_\psi(m)).$$

Steps 2 through 4 are repeated for  $m=1, \dots, M$ . The first  $B$  draws are used for burn-in and discarded. From the remaining  $M-B$  draws, every fourth draw is retained, yielding a total of  $(M-B)/4$  draws from the posterior distribution of interest. In our application, we use  $M=7200$  and  $B=4000$ , yielding a total of 800 draws for the posterior distribution.

## *B. Second Stage Estimation*

The algorithm described above provides realizations from the posterior distribution for the parameters  $\alpha_1, \dots, \alpha_J$  as well as the price and error variance terms. In this subsection we are interested in decomposing the intercepts into components that represent the observable and unobservable attributes of the recreation sites. Since the former will include endogenous attributes that ultimately provide the general equilibrium aspects of the model this decomposition is particularly important. Likewise any welfare analysis concerning site attributes (partial or general equilibrium) will require an understanding of how demand is impacted by changes in observable site quality levels.

The linearity of equation (14) suggests that the unknown parameter vector  $\gamma$  can be computed via an auxiliary regression of the intercepts on the observed quality attributes  $q_j$ , with  $\xi_j$  then computed as the residual from the regression estimates. This strategy is used in much of the empirical IO literature, and has been applied in the recreation context by Murdock (2006) and Timmins and Murdock (2007). We nest this notion within the Bayesian estimation paradigm as follows. For each of the 800 draws from the posterior distribution of  $\alpha_1, \dots, \alpha_J$  obtained in the first stage we regress the realized intercept values on the vectors of site attributes  $q_1, \dots, q_J$ . For each draw of the intercepts we therefore obtain a value for  $\gamma$ . The set of these values gives an empirical distribution that characterizes the posterior distribution for the unknown parameters  $\gamma$ , which provides measurements of how observable site attributes affect site demands.

This strategy is feasible if two conditions are met. First,  $J$  must be large enough and there must be enough variation over the  $q_j$ 's to provide sufficiently precise estimates from the linear model employed. In our application  $J=128$ , which we find to be large enough to estimate a small number of site attribute effects. Second, the observable attributes  $q_j$  must be uncorrelated

with the unobserved attributes  $\xi_j$  or instruments must be available for the (econometrically) endogenous explanatory variables. Since we are interested in examining the role of endogenous attributes we are almost certainly faced with the need to use instruments in our second stage regression. For the case of congestion, a site is likely to be heavily visited (and hence congested) if it possesses attractive attributes, some of which will not be measured by the econometrician and will therefore reside in  $\xi_j$ . This will induce (positive) correlation between the measure of congestion and the error in the second stage regression, leading to biased estimates for the role of all observable attributes on site demand.

We deal with the endogenous congestion attribute via a strategy recently suggested by Timmins and Murdock (2007) in the context of a site choice model. We define our baseline measure of congestion as the share of people who visit a particular site. For later reference it is useful to link our measure of congestion to the modeling framework described above. Thus the baseline share of people who visit site  $j$  is defined by

$$s_j^0 = N^{-1} \sum_{i=1}^N I_{ij}, \quad (37)$$

where  $I_{ij}=1$  if the person  $i$ 's notional demand for site  $j$  is positive (i.e.  $z_{ij}^* > 0$ ) and zero otherwise (i.e.  $z_{ij}^* \leq 0$ ). Our second stage estimation problem for each posterior draw from the intercepts is therefore given by

$$\alpha_j = \gamma_0 + \sum_c \gamma_c q_{jc} + \gamma_s s_j^0 + \xi_j, \quad j = 1, \dots, J, \quad (38)$$

and an instrumental variable is needed for  $s_j^0$ . We construct an instrument by estimating a binary logit model for each site, where the dependent variable is equal to one if the person visits the site and zero otherwise. The logit probability is parameterized to include all the attributes of

the site thought to be exogenous, such as travel costs and ambient water quality, and the parameter estimates used to construct predictions for each person's probability of visiting each site.<sup>7</sup> We define our instrument for  $s_j^0$  to be the average of the predictions for site  $j$  for all people in the sample. We find this measure to be reasonably well correlated with  $s_j^0$  and plausibly exogenous since it is functionally related only to exogenous variables. As Timmins and Murdock note the power of this instrument strategy depends on the degree to which the exogenous site attributes are good predictors of visitation shares. We return to this topic when discussing our results below.

### *C. Welfare Analysis*

In this section we describe how welfare analysis of changes in prices or attributes at the recreation sites proceeds. As discussed by von Haefen et al. (2004) welfare analysis in this class of models is complicated by many factors, including the need to simulate behavior under counterfactual conditions. The steps needed for analysis using the dual model are particularly challenging and involve subtle technicalities and many computational issues that at this stage of research are not fully understood. The added complexity of computing general equilibrium welfare measures compounds the difficulties. Thus we focus in this section on laying out the broad steps needed for computing welfare measures, forgoing many of the technical details in favor of a discussion of the conceptual challenges that we have solved and those that must still be examined.

The first and second stages of the estimation algorithm provide summaries via posterior means of the utility function parameters and the error variances that constitute the unknown parameters in the model. From these summaries preferences are known up to the values of the

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<sup>7</sup> In practice we estimate the binary outcomes for all sites jointly and constrain the parameters on the explanatory variables to be the same across all sites.

idiosyncratic errors – the  $\varepsilon_{ij}$ 's in our notation. Because the idiosyncratic errors are given a structural interpretation the utility function is random from the perspective of the econometrician, which implies that measures of compensating surplus for changes in prices or site attributes will also be random variables. Thus the objective of welfare analysis is to estimate the expected value of an individual's compensating surplus by integrating out the unobserved determinants of choice. This requires that we simulate values of the idiosyncratic errors for each person many times and compute the welfare measure of interest for each simulated draw. The average over the welfare measures for each simulated error draw provides an estimate of the expected welfare measure for each person.

Conditional on a simulated value of the errors for each person all components of the preference function  $V(p,y;q,\theta,\varepsilon)$  from equation (23) are known. In general we can define compensating surplus ( $CS$ ) implicitly using  $V(\cdot)$  by

$$V(p^0, y, q^0, \theta, \varepsilon) = V(p^1, y - CS, q^1, \theta, \varepsilon), \quad (39)$$

where the change is being evaluated from baseline conditions  $(p^0, q^0)$  to new conditions  $(p^1, q^1)$ . Solving for  $CS$  in equation (39) is complicated by the fact that the indirect utility function represents an endogenous regime-switching outcome: changes in  $p$  or  $q$  may induce visitors to change the pattern of sites they visit as well as the frequency, and the solution algorithm must ensure that the comparison is made for utility values that reflect the appropriate (utility maximizing) demand regime under initial and changed conditions. Von Haefen *et al.* (2004) and von Haefen and Phaneuf (2005) discuss this challenge in some detail and provide solution algorithms that are appropriate when preferences are additively separable. Because the preference function we apply in this analysis is more general a different solution algorithm is ultimately needed.

Simulating the errors and solving the consumer's problem are required for both partial and general equilibrium analysis and involve similar computation steps and techniques. The general equilibrium calculation adds an additional layer of computation, however, in that the simulated behavior at new conditions must also be used to predict the elements of  $q^1$  that are endogenously determined by the aggregation of behavior. Thus welfare calculation in this paper requires we address two methodological challenges: solving the consumer's problem when preferences are not additively separable, and predicting new levels of the endogenous attributes under changed conditions. We must also address the non-linear way in which site quality attributes enter  $V$  (recall they enter indirectly through the virtual prices for the non-consumed goods as well as directly via the intercepts) to properly compute and interpret our welfare measures.

To illustrate these challenges and describe our initial solutions we first list out the explicit steps needed to compute partial and general equilibrium welfare measures using the specification we are working with. We then describe the steps in more detail. Given the posterior means for the utility and error distribution parameters partial equilibrium welfare analysis for a single person  $i$  and a single draw of the errors involves the following steps:

- 1) Draw values of the errors  $\varepsilon_{i1}, \dots, \varepsilon_{iJ}$  from the estimated distribution for the unobserved component of utility *conditionally* such that observed levels of demand at baseline conditions are replicated for person  $i$ . That is, draw errors such that

$$z_{iC}^0 = \alpha_C^0 + \beta_{NC}\pi_{iN}^0 + \beta_{CC}P_C^0 + \varepsilon_{iC}, \quad (40)$$

where the notation follows from equation (23), superscripts '0' indicate that all price and quality variables are set to their baseline values, and  $z_{iC}^0$  denotes the observed level of visits for person  $i$  to the set of visited sites  $C$ .

2) Determine the total baseline consumer surplus *from the consumed goods* by integrating under the baseline demands in equation (52) between  $p_{iC}^0$  and  $\hat{p}_{iC}^0$ , where  $\hat{p}_{iC}^0$  is the choke price for the set of goods  $C$  at baseline conditions. Because the ordinary and compensating demand curves are the same in our case this is also the total Hicksian consumer surplus.

3) Define the counterfactual scenario to consist of new prices and/or exogenous site attribute levels  $p_{i1}^1, \dots, p_{iJ}^1$  and  $\alpha_1^1, \dots, \alpha_J^1$ , where

$$\alpha_j^1 = \gamma_0 + \sum_c \gamma_c q_{jc}^1 + \gamma_s s_j^0 + \xi_j, \quad j = 1, \dots, J, \quad (41)$$

and the  $q_{jc}^1$ 's hold new levels of the exogenous attributes. For a partial equilibrium analysis the original levels of the endogenous attributes (congestion) are maintained.

4) Determine the new demand regime by first computing the new notional demands using

$$z_{ij}^{*1} = \alpha_j^1 + \sum_{k=1}^J \beta_{jk} p_{ik}^1 + \varepsilon_{ij}, \quad 1, \dots, J, \quad (42)$$

and observe the pattern of positive and negative values for  $z_{ij}^{*1}$ . The combination of positive and negative values for each site is a candidate  $\tilde{C}^1$  for the new demand regime, which must be evaluated and updated as described below. Via the updating the new demand regime  $C^1$  is determined.

5) Determine the total baseline consumer surplus from the consumed goods at the *new demand regime* by integrating under

$$z_{iC^1}^1 = \alpha_{C^1}^1 + \beta_{NC^1} \pi_{iN^1}^1 + \beta_{C^1C^1} p_{C^1}^1 + \varepsilon_{iC^1} \quad (43)$$

between  $p_{ic^1}^1$  and  $\hat{p}_{ic^1}^1$ , where  $\hat{p}_{ic^1}^1$  is the choke price for the set of goods  $C^1$  at changed conditions.

- 6) Compensating surplus for person  $i$  for this draw of the error is the difference between total surplus at initial and changed conditions.

A few comments on this algorithm should be made. First, the algorithm focuses on obtaining use-only values from changes in the levels of attributes by only including areas under the demand curves for sites that are actually visited. A utility function approach as shown in general in equation (39) would also add surplus to the total for sites that were not visited. We have chosen to focus on use value to aid in interpretation and avoid complexities associated with general equilibrium feedbacks interacting with non-use value computation. Also, we are assuming in this algorithm that changes in observable attributes from  $q^0$  to  $q^1$  leave unobserved attribute levels unchanged. That is,  $\xi_j$  is constant for all sites across all changes. Depending on the scenario this may or may not be a realistic assumption.

For general equilibrium welfare measurement the same steps are needed, but we must also update the level of the endogenous attribute. Define step 3' for the case of general equilibrium to be

- 3') Define the counterfactual scenario to consist of new prices and *candidate* site attribute levels defined by  $p_{i1}^1, \dots, p_{ij}^1$  and  $\tilde{\alpha}_1^1, \dots, \tilde{\alpha}_J^1$ , where

$$\tilde{\alpha}_j^1 = \gamma_0 + \sum_c \gamma_c q_{jc}^1 + \gamma_s s_j^0 + \xi_j, \quad j = 1, \dots, J, \quad (44)$$

and the  $q_{jc}^1$ 's hold new levels of the exogenous attributes. For general equilibrium measurement an algorithm is needed that updates  $\tilde{\alpha}_1^1, \dots, \tilde{\alpha}_J^1$ , to  $\alpha_1^1, \dots, \alpha_J^1$ , where in this case

$$\alpha_j^1 = \gamma_0 + \sum_c \gamma_c q_{jc}^1 + \gamma_s s_j^1 + \xi_j, \quad j = 1, \dots, J, \quad (45)$$

and  $s_j^1$  is the new equilibrium proportion of people who visit site  $j$  given the changed conditions. We discuss the form that this updating takes below.

These six steps each present varying degrees of computational and conceptual challenges. Step 1 is technical but quite similar to the data augmentation stage described above. In addition the ideas associated with simulating unobserved errors consistent with observed choice are well-explained in other work. Thus in this version of the paper we do not discuss the details of this step further. Step 2 and likewise step 5 are mechanical; again, we forgo additional discussion of these steps. We do note however that the decision to rely on surplus measures computed as areas under compensated demand curves rather using expenditure or utility functions is a point worthy of further discussion, but one that is largely orthogonal to the topics directly under consideration. Thus the steps that we provide more discussion on include step 4 (determining the new demand regime) and step 3 as it relates to the general equilibrium calculation.

Consider first how we determine the new demand regime given changed conditions. We refer to equation (42) as providing a ‘candidate’ demand regime because the mapping between a set of positive and negative *notional* demands to the implied *actual* demands via the appropriate set of virtual prices does not guarantee that the resulting actual demands will be strictly positive. Thus the candidate regime may not be a member of the set of feasible demand regimes under the changed conditions. In this case a mechanism is needed to find an alternative demand regime from the set of feasible regimes (i.e. those that do not result in negative actual demands) that maximizes utility. We have not yet solved this problem formally and rely at this stage on an ad hoc updating rule. Specifically, we complete step 4 via the following:

- Observe the candidate regime  $\tilde{C}^1$  using equation (42).

- Compute the candidate actual demands  $\tilde{z}_{iC^1}^1$  for this regime and observe which, if any, of the demands are negative.
- Update the candidate demand regime by setting to ‘non-consumed’ the goods observed with negative actual demands. Label this  $C^1$  (the new regime) and use it in step 5.

This is an ad hoc updating rule in that we have not proven that it results in the utility maximizing solution under the new price and attribute conditions. Verifying this to be the case, or altering the updating strategy to find the utility maximizing solution, is an important area for subsequent research.

Step 3 is trivial in the partial equilibrium case but involves notable challenges for the general equilibrium case. An updating rule is needed that re-equilibrates the site quality indexes (the intercepts) to reflect both the new exogenous attribute levels and the new resulting congestion level. A similar challenge was faced by Timmins and Murdock (2007) for their site choice congestion application. These authors rely on results shown by Bayer and Timmins (2005) to show that their measure of congestion is the result of a unique sorting equilibrium, and that solving for new congestion levels in counterfactual experiments relies on a simple application of Brower’s fixed point theorem. Since our measure of congestion and behavioral model differ from Timmins and Murdock these results do not transfer directly. Thus at this stage of the research we are still investigating the formal properties of our equilibrium concept as well as computational methods for simulating new outcomes.

To explore this point further recall that our measure of congestion as given by equation (37) consists of the proportion of people who visit a particular site. This is a convenient metric for our model in that the related concepts of virtual price and notional demands can in principle be used to predict this proportion for the sample under any configuration of prices and

exogenous attribute levels. Consider for example the following algorithm. For iterations  $t=1,2,\dots$  complete the following steps:

a) Define  $\tilde{z}_{ij}^{*t}$  by

$$\tilde{z}_{ij}^{*t} = \tilde{\alpha}_j^{1t}(\tilde{s}_j^{t-1}) + \sum_{k=1}^J \beta_{jk} p_{ik}^1 + \varepsilon_{ij}, \quad j=1,\dots,J, \quad i=1,\dots,N,$$

$$\tilde{s}_j^{t-1} = \frac{1}{N} \sum_{i=1}^N I_{ij}^{t-1}$$

$$I_{ij}^{t-1} = \begin{cases} 1 & \tilde{z}_{ij}^{*t-1} > 0 \\ 0 & \tilde{z}_{ij}^{*t-1} < 0 \end{cases}$$

b) At iteration  $T$  define the new equilibrium congestion level by

$$s_j^1 = \frac{1}{N} \sum_{i=1}^N I_{ij}^{T-1}.$$

Understanding the formal properties of this mapping, and altering it as needed, is an important task for further research.<sup>8</sup>

The comments on solving the consumer's problem and the equilibrium simulations suggest further work is needed on welfare measurement in this model. Nonetheless we assess progress in the next section by describing results based on the IA lakes application.

## VII. Empirical Results

In this section we present empirical results for the IA lakes data set described above. We emphasize that at this stage these findings are illustrative and exploratory. Nonetheless several interesting results emerge that illustrate the importance of non-price equilibria concepts.

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<sup>8</sup> In practice it is also necessary to check that the regimes implied by  $\tilde{z}_{ij}^{*t}$  are in the feasible set and, if not, adjust using the strategy discussed above. This emphasizes the point that using predicted behavior to simulate new endogenous attribute outcomes also depends critically on the ability to solve consumers' outcomes accurately and quickly.

The model was run in MATLAB using the first stage Gibbs sampler and second stage regression decomposition to obtain an empirical representation of the posterior distribution for the unknowns in the model. The first stage is computationally intense: obtaining 7200 draws from the posterior distribution required nearly a month of run time on a new computer. There are obvious improvements in our code that can speed this process, but it is nonetheless the case that large dimension models of this type continue to be computer-time intensive. From the 7200 draws obtained we discard the first 4000 as a burn-in period and construct our empirical distribution using every fourth draw thereafter, leaving 800 draws of the 258 first stage parameters for inference and subsequent analysis.<sup>9</sup>

Table 1 contains a summary of the posterior distribution for the own- and cross-price parameters, and the top half of Figure 1 provides a histogram of the full marginal posteriors for both these parameters. The price coefficients can be directly interpreted as the marginal effects of price changes on the notional demands, but their interpretation as related to the actual demands is more complex since the parameters enter the demand equations non-linearly. Nonetheless the signs and ratios of the posterior means and standard deviations seem reasonable. We find that own price effects are two orders of magnitude larger than the cross price effects, suggesting there may be little cross-site substitution on average. We note that restricting all cross-price effects to be equal likely masks substantial heterogeneity among specific lakes and an important task for further research is to explore specifications that remain tractable in the number of parameters but also allow a greater deal of price substitution to appear.

Tables 2 and 3 present different strategies and results for decomposing the intercepts into observable and unobservable site attributes, and the bottom half of Figure 1 provides histograms

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<sup>9</sup> Multiple MCMC chains were run simultaneously on different computers during the month of run time to gauge convergence of the chains via comparisons. We found strong evidence that burn-in was easily achieved by the 4000<sup>th</sup> iteration.

of the full marginal posterior distributions for the parameters on the attributes for the specification in Table 3. Once the empirical distribution for the intercepts is obtained in the first stage it is computationally fast and straightforward to investigate different specifications for the second stage. We experimented with different water quality attributes as second stage explanatory variables and settled on the use of two: secchi disk measurement and ambient levels of chlorophyll. These two measures of site quality are potentially attractive in that their effects are observable to visitors. Secchi readings reflect observable water clarity (assumed to be a positive attribute of lakes) while chlorophyll reflect visible algae and weed growth, which are correlated with nitrification. We stress nonetheless that other specifications may be preferred. We are however degrees-of-freedom limited. The second stage regressions exploit variation over sites, so estimates are based in our case on only 128 observations.

The results are illustrative of both the difficulties and importance of accounting for endogenous attributes such as congestion. Table 2 contains our straw-man results. Here he have naively included an obviously endogenous variable (proportion of people visiting each site) in the equation and used OLS to estimate the parameters. We find a negative effect on congestion but have no resolution on our site quality estimates. In contrast the results in table 3 are much more promising and intuitive. We find a large and negative coefficient on congestion and a solidly significant (from a classical perspective) of the correct sign on chlorophyll. The sign on secchi is appropriately positive but at best marginally significant. From this we cautiously conclude that our instrument strategy is viable, and that congestion matters – probably more than exogenous attributes such as ambient water quality and perhaps as much as own price effects.

This finding is very similar to Timmins and Murdock (2007), who find using their preferred instrument strategy large and significant disutility from congestion.<sup>10</sup>

A primary objective of estimating the parameters of the structural model is to examine both partial and general equilibrium welfare measures. To illustrate the capabilities of the model in this dimension we again consider four counterfactual scenarios, each designed to illustrate welfare measures of potentially different types. The scenarios are:

Scenario 1: Close nine sites representing the most heavily visited lakes in each of nine regions of Iowa.

Scenario 2: Close nine sites representing moderately visited sites in each of nine regions of Iowa.

Scenario 3: Improve water quality throughout the state such that all lakes obtain at least the rating of ‘good water quality’. According to technical documents this corresponds to a minimum secchi reading of 2.17 meters and maximum chlorophyll reading of 8.26ug/l. This scenario involves improvements at for at least 114 lakes.

Scenario 4: Improve a set of seven Iowa Department of Natural Resources ‘target lakes’ to water quality conditions given by a minimum secchi reading of 5.7 meters and maximum chlorophyll reading of 2.6ug/l.

The first scenario is major in that it involves the loss of the nine primary lakes in the state, while the second is arguably minor in that the lakes are minor regional facilities. In both cases we proxy the loss of the sites by setting travel costs above the choke prices for all visitors in the sample. The third and fourth scenarios are minor but widespread and major but localized, respectively.

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<sup>10</sup> Table 5 presents the posterior means and standard deviations for the sites-specific intercepts and variance terms. These are interesting only to note the heterogeneity across sites in both sets of parameters.

Point estimates for compensating surplus measures are shown in Table 4, with sample mean and median partial equilibrium estimates aligned on the left and the general equilibrium measures aligned on the right. The estimates are seasonal per person measures; for a rough per trip measure one could normalize by the mean or median total annual trips taken (11.46 and 7 trips, respectively). We find plausible estimates for the partial equilibrium estimates in all cases. The loss of the nine popular sites leads to a large mean welfare loss of over \$600 per person per season. The much smaller median loss of nearly \$68 per person suggests the mean is skewed by individuals with high valuations for the lost sites. This is sensible in this model and matches our intuition: people who do not visit the lost sites, or do so only infrequently, do not suffer a surplus loss when the sites are eliminated. This is also seen in scenario 2, where the sample average loss from closing 9 moderately popular sites is nearly \$52 per angler and the median is zero. Closing the less important sites impacts less than half the people in the sample, suggesting that most suffer no surplus loss from the closures.

Scenarios 2 and 3 examine quality improvements of different intensity and spatial extent. Here we find that smaller, more widespread quality improvements have a larger welfare impact (sample mean and median of approximately \$208 and \$147, respectively) than their localized but larger counterparts (\$50 and \$17). In both cases the positive median suggests benefits are spread throughout the sampled population.

The general equilibrium welfare measures also seem plausible for all scenarios, though we caution that these measures are preliminary in that further research is needed to understand the properties of re-equilibrating algorithm. For the site loss scenarios in particular it is unclear that a new sorting equilibrium is achieved; evidence of this is stronger for the quality changes. Nonetheless some intuition emerges. Using the means in scenario 1 we find general equilibrium

welfare losses that are 15% larger than their partial equilibrium counterpart. Similarly for scenario 2 we find general equilibrium losses that are 11% higher. This reflects the fact that the site closures cause a direct welfare effect via the lost choice alternatives as well as an indirect effect on the remaining sites via increased congestion. The intuition for the direction of the general equilibrium effect is less clear for the quality changes. If improvements are made to moderately popular lakes, and by attracting visitors from more popular lakes decreases congestion, the general equilibrium effect may be larger. In contrast improvements at currently congested sites that cause more people to go to these sites may lead to smaller general equilibrium welfare improvements. For our scenarios we find general equilibrium effects that suggest smaller improvements when re-sorting is accounted for. In particular the welfare effects are less than half as large when the indirect effect of changes in congestion at the lakes is explicitly accounted for.

## **VIII. CGE Modeling**

Our objective for the CGE component of the analysis is to explore the analytics of the problem we are considering and to begin to understand more generally the situations in which non-price feedback effects might be empirically important. The basic setup of the model follows the general modeling framework described by equations (1), (2), and (3). To execute the CGE experiment we must consider welfare calculation, functional forms, calibration, and counterfactual experiments. We describe each of these in turn.

### *A. Welfare Calculation*

In general computing compensating variation within the numerical model is straightforward. It is possible to form first-order conditions for each consumer's expenditure minimization problem by constructing the relevant Lagrangian function and taking the partial

derivative with respect to the three classes of choice variables:  $z_{ij}$ ,  $x_i$ , and the Lagrange multiplier  $\lambda_i$ . Given benchmark utility levels based on the initial solution to the equilibrium model and the resulting quality levels from the simulation of some policy experiment, we then can solve each individual expenditure minimization problem as the solution to the following system of equations:

$$\begin{aligned}
p_{ij}^1 - \lambda_i \frac{\partial U_i}{\partial z_{ij}} &\geq 0, z_{ij} \geq 0, z_{ij} \left( p_{ij}^1 - \lambda_i \frac{\partial U_i}{\partial z_{ij}} \right) = 0 \\
1 - \lambda_i \frac{\partial U_i}{\partial x_i} &\geq 0, x_i \geq 0, x_i \left( 1 - \lambda_i \frac{\partial U_i}{\partial x_i} \right) = 0 \\
U_i^0 - U_i(z_i, x_i; Q^1, p^1) &\geq 0, \lambda_i \geq 0, \lambda_i \left[ U_i^0 - U_i(z_i, x_i; Q^1, p^1) \right] = 0,
\end{aligned} \tag{46}$$

where  $U_i^0$  is the initial utility level of individual  $i$  from the benchmark equilibrium model and  $Q^1$  and  $p^1$  describe vectors of quality attributes and travel costs that result from the equilibrium model after the proposed policy intervention. Repeatedly solving this problem based on draws from the underlying error distribution yields average *CV* calculations.

In the numerical simulations that follow, we calculate *CV* based on changes in consumer surplus using changes in the areas under the demand curves for each site and individual. The linear form of the demand functions makes this computationally tractable. However, the technique described above is potentially useful for any application in which the analyst can write down the consumer's expenditure minimization problem.

### *B. Numerical Model*

The numerical model is based on the same Iowa Lakes application described in the estimation section of the paper. Calibrating the simulation model to the same database allows us to verify the observed benchmark equilibrium in the data, calculate welfare measures for counterfactual experiments which can be compared directly to those produced by the statistical

model, and make predictions on the sensitivity of general equilibrium effects to some of the key assumptions in the specification of the model.

The set of sites and quality characteristics is the same as in the estimation experiment. The one difference between the dimensionality of the dataset used in the application and the experiment described here is that we do not represent the full population of individuals in the numerical model. Solving the linear complementary problem described in equations (48) and (50) or (51) below using the full sample of individuals (749) and sites (128) from the dataset would produce a system of  $749 \times 128 + 128 = 96,000$  equations/inequalities. While solving such a linear system is feasible, it would take a prohibitively long time on the hardware available to us for this project. Our compromise is to solve and present the results of the simulation models based on a sub-sample of 300 randomly selected individuals from the data. The results presented in the main body of the paper are based on one such sub-sample. However, we also describe the results of sensitivity analysis to this sampling procedure in materials available upon request. We find that the variance in our results across different sub-samples is quite small.

### Model Specification

Following the logic of the estimation strategy, the demand system is based on a dual representation of consumer behavior. The outcome of the consumer choice process is represented by an indirect utility function  $H$ . Our main results are based on a linear demand system, which corresponds to the following form for  $H$ :

$$H(\pi_i) = y - \left[ \sum_j \left( \alpha_{ij} + \sum_m \gamma_m q_{mj} \right) \pi_{im} + \frac{1}{2} \sum_j \sum_k \beta_{jk} \pi_{ij} \pi_{ik} \right], \quad (47)$$

where  $\pi_i$  is a vector of virtual prices that rationalizes the system of notional demands to take the non-negativity constraints on site demand into account. Thus, the virtual prices associated with

demand for each site are determined by the following optimality conditions obtained via an application of Roy's Identity:

$$\frac{-\partial H_i / \partial \pi_{ij}}{\partial H_i / \partial y_i} = z_{ij} = \alpha_{ij} + \sum_m \gamma_m q_{mj} + \sum_k \beta_{jk} \pi_{ik} \geq 0 \perp \pi_{ij} \leq p_{ij}. \quad (48)$$

Congestion is assumed to make its contribution to utility separately from the other quality characteristics in the data and these characteristics are assumed to be exogenous such that

$$q_{mj} = q_{mj}^0 \quad \forall m \neq 1, \quad (49)$$

where  $m=1$  denote our congestion measure and  $q_{mj}^0$  is the level of attributes given by the data for each site in the sample.

One advantage of modeling both the intensive and extensive margins of choice in our recreation demand application is that it allows us to examine how the *intensity* of visitation to different sites affects congestions. This is in contrast to RUM-based applications (e.g. Timmins and Murdock, 2007) in which congestion is measured by the share of visitors who go to each site. Based on this, we consider two different specifications for the transmission function. In the first, congestion is defined as the aggregate demand share at site  $j$ :

$$q_{1j} = \frac{\sum_i z_{ij}}{\sum_i \sum_k z_{ik}}. \quad (50)$$

This specification mimics the approach used to produce both the welfare calculations from the statistical model in this paper and the results reported in Murdock and Timmins(2007). The second specification defines congestion as simply the total number of visitors to site  $j$ :

$$q_{1j} = \sum_i z_{ij} \quad (51)$$

This specification will obviously take the impact of the intensity of visitation on congestion into account in our simulation results.

Following the discussion from above the equilibrium model consists of first-order conditions based on (48) and the quality conditions in either (50) or (51). The GAMS code for the model specifications and solution routines is available upon request. From a computational perspective, this type of problem is easily characterized as a mixed complementarity problem (MCP). The GAMS mathematical modeling software combined with the PATH MCP solver provides a robust way to solve large-scale MCP problems.<sup>11</sup>

### Equilibrium Effects of Congestion

Writing out the algebra for the congestion transmission function on the right-hand side of (51) provides some intuition on the equilibrium implications of the negative feedback between visitation and congestion. Re-writing, (51) we have

$$q_{1j} = \sum_i \left( \alpha_{ij} + \sum_m \gamma_m q_{mj} + \sum_k \beta_{jk} \pi_{ik} \right). \quad (52)$$

Collecting the  $q_{1j}$  terms on the left-hand side of the equation and solving for  $q_{1j}$ , we have

$$q_{1j} = \frac{\sum_i \left( \alpha_{ij} + \sum_{m \neq 1} \gamma_m q_{mj} + \sum_k \beta_{jk} \pi_{ik} \right)}{1 - I\gamma_1}. \quad (53)$$

Note that the term in the denominator is greater than one if congestion has a negative effect ( $\gamma_1 = \gamma_{congest} < 0$ ) on visitation. Now consider the effect of a policy change on the equilibrium level of congestion. If, for example, some aspect of water quality is improved at site  $j$  the direct effect on the level of congestion (measured by the numerator of (53)) causes congestion to rise.

However, the negative feedback effect caused by this increase in congestion leads to a

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<sup>11</sup> Michael Ferri's documentation of the PATH solver provides a concise description of the MCP class of problems and the solution algorithm used by PATH. See the solver documentation page on [www.gams.com](http://www.gams.com)).

subsequent decrease demand for site  $j$ . This in turn reduces the level of congestion at the site, and so on. The term in the denominator of (53) describes the net result of this adjustment process on the level of congestion, measured as the factor by which the direct effect of the increase in congestion is attenuated by the general equilibrium effects. Furthermore, notice that the sensitivity of the feedback effect depends directly on the magnitude of the  $\gamma_{congest}$  parameter – when the demand response to congestion is stronger, the feedback effect is stronger.

For this logic to tell the whole story, it must not be the case that this adjustment induces changes in the virtual prices at zero-visit sites. If there were systematic changes in these prices across individuals in response to the policy, this could complicate the equilibrium adjustment story described here. Nonetheless, the basic feedback mechanism should be clear from our discussion.

### *C. Calibration*

The numerical model is calibrated to the same Iowa Lakes dataset used in the estimation routine. Broadly, the idea is to match the model to the observed demands in the benchmark data exactly in equilibrium, and to match the price and quality responsiveness of the individual demand functions to the point estimates of the demand coefficients in the estimation routine. The point estimates that are used are:  $\beta_{jj}=-0.045$  for all  $j$ ,  $\beta_{kj}=0.0002$  for all  $j$  and  $k$  not equal,  $\gamma_{secchi}=0.084$ ,  $\gamma_{chloro}=-0.006$ , and  $\gamma_{congest}=-54.34$ . Because the  $\gamma_{congest}$  parameter is a key determinant of the positive and normative consequences of congestion in the model, we also perform sensitivity analysis in which we compare the outcomes of simulation experiments in which the model is calibrated to values of  $\gamma_{congest}$  that are one-half and twice the value of its point estimate from the estimation results. When the “totals” specification of congestion described in

equation (51) is used,  $\gamma_{congest}$  is re-scaled by dividing by the total number of visits to all sites in the benchmark dataset.

### Calibration of Random Parameters

The basic premise of the estimation strategy is that there are unobserved components of individual tastes due to the fact that non-negativity constraints on demands are binding at many of the sites in the benchmark dataset. Thus, the form that the demand functions take in the specification of the linear demands in the estimation model is:

$$z_{ij} = \alpha_{ij} + \sum_m \gamma_m q_{mj} + \sum_k \beta_{jk} \pi_{ik} + \varepsilon_{ij} \geq 0 \perp \pi_{ij} \leq p_{ij} \quad (54)$$

where the difference between (54) and (48) is the addition of an error term  $\varepsilon_{ij}$  in (54). In order to calibrate the simulation model in a manner consistent with this logic, we must allow for the fact that  $\tilde{\alpha}_{ij} \equiv \alpha_{ij} + \varepsilon_{ij}$  may take on a range of values that are consistent with replicating the benchmark dataset. Consider two possible scenarios. For a given individual, the non-negativity constraint for one of her demand functions may be weakly binding in the sense that a marginal reduction in the travel cost to that site would induce visits, or it may be strongly binding in which case even making the site substantially more attractive from the individual's perspective would still yield zero visits. Simulating policy experiments and calculating the welfare consequences under these two different scenarios would obviously lead to different results for this individual. It is also possible that systematic differences along these lines might affect equilibrium outcomes at the aggregate level through their affects on the congestion mechanism.

One approach to calibrating these  $\tilde{\alpha}_{ij}$  parameters is to take draws from the posterior distribution of these parameters described by the estimation results. By repeating our counterfactual experiments for different sets of draws from these distributions, we can describe

the distribution of the outcome measures that we are interested in such as equilibrium visitation and congestion levels at different sites and welfare changes for different types of individuals.

While there is not a unique mapping of the  $\tilde{\alpha}_{ij}$  terms to observed demands, the conceptual logic of the virtual prices described above does impose some structure on the calibration problem.

Specifically, it pins down the value of the virtual price so that  $\pi_{ij}=p_{ij}$  if the benchmark number of visits that an individual takes to site  $j$  is non-zero. Thus

$$\tilde{\alpha}_{ij} = \tilde{z}_{ij} - \left[ \sum_m \gamma_m \tilde{q}_{mj} + \beta_{jj} p_{ij} + \sum_{k \neq j} \beta_{jk} \pi_{ik} \right] \geq 0 \quad \forall j \text{ s.t. } \tilde{z}_{ij} > 0, \quad (55)$$

where  $\tilde{z}_{ij}$  and  $\tilde{q}_{mj}$  are the benchmark levels of site visits and attribute levels observed in the data for each site.

For site demands that are zero in the benchmark data, we draw the  $\tilde{\alpha}_{ij}$  randomly according to the posterior distribution for these terms. These draws imply specific realizations of the virtual prices associated with each demand because they must replicate the observed demand of zero given  $\tilde{\alpha}_{ij}$  and the other virtual prices in the demand system. Thus,

$$\pi_{ij} = - \left[ \sum_m \gamma_m \tilde{q}_{mj} + \sum_{k \neq j} \beta_{jk} \pi_{ik} \right] / \beta_{jj} \geq 0 \quad \forall j \text{ s.t. } \tilde{z}_{ij} = 0. \quad (56)$$

The calibrations of the free  $\alpha$  parameters (those for the positive demand sites) and the virtual prices (i.e., those for the zero-demand sites) are characterized by the solution to the system of equations defined by (55) and (56). For a given individual, if  $N$  is the number of non-zero sites demands and  $Z$  is the number of zero site demands, this strategy produces a system of  $N + Z$  equations and  $N + Z$  unknowns –  $N$  of the  $\tilde{\alpha}$  terms and  $Z$  of the  $\pi$  terms.

The solution to this system of equations is not, however, guaranteed to produce a set of virtual prices that do not violate the condition of the model that these prices be less than the

observed travel cost prices to each site. Therefore, the full calibration routine involves sequentially solving this system of equations, checking to insure that the resulting virtual prices do not violate their upper bounds, taking new draws from the  $\tilde{\alpha}$  distributions for those demands that do not violate this condition, and solving for the new virtual prices until no such violations occur.

#### *D. Counterfactual Experiments*

The purpose of the simulations described in this section is to explore the sensitivity of the welfare calculations to the form of the equilibrium model. In particular, we investigate the extent to which the welfare calculations are sensitive to two assumptions of the model specification: the intensity of the individual site demand response to the level of congestion at that site, and the form of the congestion measure itself.

The welfare results that we report are based on same counterfactual scenarios used in the empirical model, which we repeat here:

- Scenario 1: Close nine sites representing the most heavily visited lakes in each of nine regions of Iowa.
- Scenario 2: Close nine sites representing moderately visited sites in each of nine regions of Iowa.
- Scenario 3: Improve water quality throughout the state such that all lakes obtain at least the rating of ‘good water quality’. According to technical documents this corresponds to a minimum secchi reading of 2.17 meters and maximum chlorophyll reading of 8.26ug/l. This scenario involves improvements at 114 lakes.
- Scenario 4: Improve a set of seven Iowa Department of Natural Resources ‘target lakes’ to water quality conditions given by a minimum secchi reading of 5.7 meters and maximum

chlorophyll reading of 2.6ug/l. These quality characteristics correspond to the cleanest among the 129 lakes in our choice set.

For each of the policy scenarios that we have outlined, we simulate the calibrated model under the assumption that the  $\gamma_{congest}$  parameter takes on values of one half, one, and two times the value of the point estimate for this parameter from the estimation results. When  $\gamma_{congest}$  is twice (half) the value of the point estimate, a marginal increase in congestion at a site will be twice (half) as influential in discouraging site demand, and so we would expect to observe magnified (diminished) impacts of congestion in the full equilibrium response to the policy counterfactuals.

The benchmark assumption in this analysis is that the level of congestion at a given site is a function of the total number of visits to that site in equilibrium. In practice, we have no strong prior (or strong guidance from the literature) on what form this externality transmission function should take, however. To explore the sensitivity of the model to this assumption and to link the simulation results to the empirical experiments in the paper, we also simulate the model under the assumption that congestion depends only on the share of visitors to each site. The obvious implication of this alternative specification for congestion is that the aggregate increases or decreases in demand will not have an effect on the damages associated with congestion.

Table 6 reports on the welfare results for the core simulations. The rows of the table list the four different policy scenarios. The columns of the table indicate whether the welfare measure is a partial equilibrium (PE) welfare measure, in which the level of congestion at each site is held fixed at the benchmark level, or a general equilibrium (GE) measure in which congestion is determined as an endogenous part of the demand system, or the percentage difference between the PE and GE measures. For each set of these measures, the columns

indicate the assumed value for the parameter in the model that determines the intensity of the individual demand response to an increase in the level of congestion at a site – either half, once or twice the value of the point estimate. Table 7 duplicates the results for the core model run ("totals" specification and middle  $\gamma_{congest}$  estimate) and compares these results with the results of a comparable model in which the "shares" specification of the congestion transmission function is used.

Turning to Table 6, first consider the overall pattern of the welfare changes for the different counterfactual scenarios where  $\gamma_{congest}$  is calibrated to the value of the point estimate from the data. In both of the scenarios that involve shutting down lakes to recreation, the sign of both the PE and GE welfare effects is negative, reflecting the fact that consumers are worse off as a result of the loss of these sites. The magnitude of the loss is larger for policy scenario #1 than for #2 because the former involves shutting down the most popular sites in different regions of the state and scenario #2 involves shutting down only moderately visited sites. Similarly, both of the proposals that increase water quality characteristics lead to overall welfare improvements, with larger improvements associated with scenario #3 than #4.

The GE welfare measures, which take into account the adjustments in site congestion levels in the wake of the demand responses, are not significantly different from the PE estimates in magnitude for the policy scenarios that involve shutting down sites (scenarios #1 and #2), suggesting that the effect of the re-sorting induced by the policy change does not have important welfare implications. The difference between these estimates is less than 2% for the core simulations. However, the differences are more substantial for both of the water quality improvement scenarios – by roughly 40% in both cases. Turning to the results of the sensitivity analysis with respect to the  $\gamma_{congest}$  parameter, we see that the effect of increasing the

responsiveness of demand to congestion has the hypothesized effect – the differences between the PE and GE welfare measures that we observe in the core simulations are magnified.

It is relatively easy to account for the results of the quality-change experiments. The direct effect of the policy is to increase lake quality levels at some or all site in the model. This leads to welfare gains to visitors. The general equilibrium response to increased quality is higher congestion at these improved sites which tends to offset the welfare gains from quality improvements. Thus the GE estimates are smaller than the PE estimates, and the difference between the two is an approximate measure of the effects of congestion described by the logic in equation (53).

The fact that the role of the congestion effect is not the same between the quality-improvement and site-shutdown scenarios speaks to the range of outcomes admitted by this type of model. Table 7, which describes the results of the comparison between the "shares" and the "totals" specifications of the model for the same policy counterfactuals, is useful in explaining this behavior. Again, the rows of the table describe the different policies, and the columns describe PE and GE welfare measures for each of the two different versions of the model.

The two specifications lead us to somewhat different conclusions about the general equilibrium implications of the policy scenarios. In the shares specification, the policies that were left unchanged by congestion effects under the totals assumption (site shutdown scenarios) lead to welfare losses in general equilibrium that are roughly 6% larger than the PE welfare estimates. Perhaps more striking is the disappearance of the congestion effects in the quality-improvement scenarios, which lead to roughly 40% overstatement of the benefits in the PE measures under the totals specification of the model.

While in principle many effects contribute to determining the equilibrium outcomes we report, an important determinant of the differences between these two models lies in the effect of the policy change on the intensity of the congestion externality at the different sites after individuals are allowed to adjust their visitation patterns. First, consider the site shutdown policies. Site shutdown is equivalent to a dramatic increase in the price of visiting the affected sites. This leads to welfare losses for those individuals who were visitors before the policy change. The resulting PE cross-price effect leads to higher visitation at all other sites in the model. In the GE adjustment process, this shift to higher congestion levels at all other sites and lower (zero) congestion at the sites that were affected by the policy. The increase in congestion at the substitute sites has a direct, negative impact on consumer welfare along the lines of the effects described for the quality-improvement scenarios.

However, as the discussion of the demand system below demonstrates, the reduction in the level of congestion at sites that have been shut down also affects demand. Despite the fact that an individual no longer visits such a site after it has been shut down, her virtual price for that site remains a function of the characteristics of that site and a determinant of demand (and consumer surplus) at other sites. Because of this, a decrease in the level of congestion at that site will tend to increase a consumer's virtual price for that site. The increase in this price tends to increase the value of visitation to substitute sites, thus conferring welfare gains and expanded visitation at these alternative sites. This explains why GE welfare losses in the shutdown scenarios are less damaging than one might expect in the totals specification of the model.

The severity of the increase in congestion at the substitute sites is a direct function of the form of the congestion transmission function that we assume. Because the congestion measure in the totals specification registers the fact that total visitation increases or decreases after these

policies, it captures congestion effects that are based both on substitution across sites (as in the site shutdown scenarios) and those that are based primarily on increased overall visitation (as in the quality-improvement scenarios). The same is not true of the shares model, where the measure of congestion is not sensitive to the aggregate level of visitation.

The PE and GE comparisons for the quality-improvement scenarios across the totals and shares specifications reflect this fact. Because these scenarios tend to involve less substitution and more aggregate change in the level of visitation, the large congestion effects that we observed in the totals model disappear under the shares assumption. This is particularly evident in scenario #3, in which the policy involves an improvement to a large number of lakes in the sample. This tends to minimize the substitution effects across sites.

Overall, the simulations produce results both anticipated and unanticipated by our conceptual modeling. Strong congestion effects which conform to the mechanism described in (53) are apparent in the core simulation results based on the totals model, and their importance varies in the expected manner with the assumed value of the congestion parameter. However, the fact that the reduction of congestion at sites that are shutdown in scenarios #1 and #2 offsets the negative impact of congestion is a surprise.

The effect of changing the form of the transmission function from the totals to shares specification also yields interesting results. In particular, whether or not this function captures the intensity of visitation and whether or not the policy in question results in significant substitution across sites are both important determinants of the magnitude of the congestion effects. Comparing these two specifications was useful diagnostically in this modeling exercise. Perhaps more importantly, it leads us to conclude that obtaining the form for these functions in different policy applications should be a priority for future work in this area.

## **IX. Discussion**

Our objective in this paper has been to explore the notion of non-price equilibria and feedback effects in non-market valuation using both CGE and econometric methods. We have specifically focused on defining in general the concept of non-price equilibria, investigating the circumstances under which they might be empirically important for non-market valuation, and exploring how to measure the effects if they exist. As we stressed at the outset this paper is the initial rather than the final step in this direction, and it leaves much unresolved. Nonetheless several insights have emerged along with promising leads for continuing this line of research. To conclude the paper we summarize the findings, lessons, and speculations that have emerged from this project and identify specific areas for further research, placing them in the context of the ‘frontiers’ theme of this paper.

While we have categorized the types of non-price equilibria that may arise using the concepts of simple and complex sorting equilibrium it is difficult to say much more of operational value that is not context specific. Nonetheless there is a fairly general set of issues that emerge from our experience and need to be resolved in any application of this type. These include, for example, specifying the mechanism through which aggregate behavior translates into an endogenous attribute, choosing a tractable parameterization for this mechanism (i.e. the transmission function), and determining the computational and conceptual properties of the equilibrium associated with the transmission function. We have provided examples of these issues as they relate to congestion in a fairly general model of seasonal multiple site recreation demand.

To investigate congestion in the recreation context we have taken the fairly unusual step of using both CGE and econometric modeling approaches. Our intent, and that of the workshop organizers who suggested and helped assemble the research team, was to consider the issue from

different perspectives using what we hoped would be complementary tools. This proved to be a good decision and gives an example of the scientific value of coordinated experiments and replication exercises.<sup>12</sup>

The CGE model functioned as a laboratory which, when calibrated to our application, allowed us to explore how partial and general equilibrium results are sensitive to the magnitude of the congestion effect and the parameterization of the transmission function. Both of these dimensions of the problem turned out to be important. The flexibility of this approach, both in decomposing the results of counterfactual scenarios and performing sensitivity analyses, makes it a valuable tool when the modeling exercise demands that we represent multiple, interacting channels of influence. The role for this type of analysis becomes even clearer when one considers that we have chosen a relatively simple example of a non-price equilibrium as our application. A necessary precursor to any experiment in which we expect to make quantitative inferences from models with complex interactions between market and non-market activities is a period of intuition-building. The CGE framework is made for this type of activity.

Beyond this, the marriage of CGE and techniques from non-market used in this experiment seem to confer ancillary benefits in a number of areas. The calibration procedure used in the CGE exercise incorporates information on unobserved heterogeneity in consumer tastes from the empirical model in a way that is not standard practice in the CGE literature, where individual-level variation is typically subsumed by the preference specification for a representative agent. Furthermore the solution techniques employed in contemporary CGE

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<sup>12</sup> As a second example that is also related to the concept of general equilibrium in non-price space see the papers by Marty Smith and Larry Crowder (2005), and David Finoff and John Tschirhart (2005) that were commissioned for the workshop *Linking Economic and Ecological Models for Environmental Policy Analysis*, Santa Fe, NM, April 2005. Smith and Crowder consider the problem of integrating fish population and effort models using a reduced form approach while Finoff and Tschirhart take a structural approach. Both papers deal with the concepts of equilibrium fish populations and catch effort and are therefore directly relevant to our discussion.

modeling appear to have promise as a strategy for addressing the challenges of welfare analysis in corner solution models as described above and by von Haefen et al. (2004). This is an area of research worthy of additional pursuit.

The evidence from the empirical exercise also supports the notion that general equilibrium effects may be important in the application we consider. Similar to Timmins and Murdock (2006) we find negative effects of site congestion that appear to be substantial. While it is difficult to consider the congestion effect independently from other attributes it seems fair to extrapolate from our findings that congestion plays a role much larger than cross price effects, somewhat larger than direct water quality effects, and perhaps as great as own price effects in our model and application. These statements need to be conditioned, however, on the form of the congestion effect included. In using the proportion of people who visit a site as our measure of congestion we have ignored two potentially important determinants of that attribute: the intensity of use as well as the timing of use. Including these involves a re-parameterization of the model and transmission function, which are obvious directions for further research in both CGE and econometric settings.

The welfare calculations from the empirical model are sensible and intuitive and suggest that general equilibrium welfare measures can differ from their partial equilibrium counterparts in ways that have policy relevance. We again, however, must add caveats to this statement. The computational steps needed to compute both partial and general equilibrium welfare measures in this model are not yet fully understood and are the subject of ongoing research. Still, we have enough confidence in the results to conclude that feedback effects exist and for some counterfactual scenarios will cause divergence between the two welfare measures. The size and direction of the divergence will depend on the specifics of the scenario. This finding is also

supported by the CGE modeling. Thus the evidence directly supported or implied by our modeling and results is that congestion matters, its effect is large enough to cause divergence between general and partial equilibrium welfare measures, and frameworks exist or are in development that can empirically measure the size and policy significance of these divergences.

In addition to this direct evidence several lessons and observations have emerged from the experience of carrying out this project. Perhaps most valuable is the appreciation gained of the combined technical and conceptual challenges that are inherent in addressing this type of problem. Our optimistic sense is that the computational challenges will not be limiting. The algorithms undeveloped at this stage are solvable either with brute computer force or clever programming and numerical analysis (probably in the end both). Perhaps more interesting will be the conceptual issues. We have found it challenging to rely on intuition to gauge the ‘reasonableness’ of results when feedback effects are present in the models. More to the point, it is not clear how to best carry out robustness and reality checks. For example, we discussed above that the transmission function used in our empirical analysis is restrictive in that it ignores the timing and intensity of site usage. How might we examine how robust our results are to this obvious simplification? More generally, how can researchers do sensitivity analysis on the numerous parametric stands that will need to be taken in order to specify a transmission function for the general equilibrium attribute? More subtly, how can we gauge the impact of ‘limit effects’ or the potential need for non-linear/discontinuous transmission functions? In our model, for example, closing a recreation site drives congestion for that site to zero. The zero congestion is mechanically transmitted to the rest of the model as an improvement in a site attribute – something that does not make intuitive sense as the result of a site closure. A less extreme examination of limit effects might be concerned with the range of exogenous data values that

provide good endogenous attribute predictions via the transmission function versus those that extrapolate beyond the range of the function's ability to accurately predict. While this may seem a minor point it will be critical for gauging the accuracy of large-scale policy counterfactuals – exactly the type of policy scenarios for which we might expect re-equilibration to matter.

To these challenges we can add the fact that we have thus far only considered a simple sorting equilibrium application. Complex sorting equilibria add an additional layer of modeling to the mix, requiring that the analyst also take a stand on the form of the natural or other process that simultaneously (with behavior) determines the level of the endogenous attribute. Finally, we might wonder if multiple and interacting endogenous attributes are at play: for example, it may be that both congestion *and* fishing catch rates are endogenously determined and that these factors might interact in distributing peoples' choices. This notion presents conceptual, computational, and econometric challenges that could fill a career.

These challenges notwithstanding, our experience in this project has caused us to be optimistic about our ability to ultimately provide policy relevant inference on general equilibrium welfare measures arising from non-price equilibria. The work reported on represents progress in several technical areas and suggests avenues for further research in these areas. For example, the Lee and Pitt model as applied here is at the forefront of modeling the demand for quality differentiated goods. Its framework provides the potential for capturing rich parametric and stochastic substitution between commodities while allowing for non-additively separable preferences. Further research in this area could examine specifications that relax the independent errors assumption as well as increase the flexibility with which we characterize cross-price and income effects. The latter would involve a specification that moves away from the incomplete demand system approach in favor of explicitly modeling expenditures on outside goods as a

substitute commodity – an approach that might also allow the inclusion of non-users in the model. Econometrically, the approach we have presented draws on three strands of literature: the classical use of Bayesian simulation methods, the true Bayesian econometric paradigm, and the tradition in empirical industrial organization of using two stages to deal with unobserved product attributes and endogenous attributes. Our exploitation of these ideas to date has been mainly informal and intuitive. There are obvious gains to a more careful (and formal) integration of these three econometric approaches in this class of problem. One particular avenue would be the use of informative priors to mitigate the second stage estimation degrees of freedom problem that will be present in most recreation applications, where it is unusual to have data on more than 100 recreation sites.

Progress in these technical areas will allow more focused examination of the conceptual challenges we have identified. Joint calibration and econometric studies seem well suited for systematically examining many of these issues. Likewise, progress in other literatures (such as bio-economic modeling of spatial fisheries – see Sanchirico, Smith, and Wilen (2007), also commissioned for this workshop) will provide insights and techniques that will be directly applicable to studying non-price equilibria in non-market valuation.

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Table 1: Summary of Selected Parameters from Posterior Distributions: 1<sup>st</sup> Stage<sup>a</sup>

Parameter	Posterior Mean	Posterior Std. Deviation	Mean/Std. Deviation	Posterior Median
$\beta_{\text{own}}$	-0.046	0.0015	-30.596	-0.0459
$\beta_{\text{cross}}$	0.0002	0.000	20.10	0.0002

<sup>a</sup>Calculated using 800 simulated draws from the posterior distribution. Posterior summaries for intercepts and variances shown in a subsequent table.

Table 2: Summary of Selected Parameters from Posterior Distributions: 2<sup>nd</sup> Stage OLS Decomposition<sup>a</sup>

Parameter	Posterior Mean	Posterior Std. Deviation	Mean/Std. Deviation	Posterior Median
$\gamma_0$	-5.435	0.2015	-26.96	-5.429
$\gamma_{\text{secchi}}$	0.0391	0.0862	0.453	0.0411
$\gamma_{\text{chlorophyll}}$	0.0013	0.0017	0.744	0.0014
$\gamma_{\text{congestion}}$	-13.34	2.14	-6.22	-13.32

<sup>a</sup>Calculated via OLS regressions of each first stage draw of the intercepts on the site characteristics. Summaries are calculated from the resulting 800 sets of 2<sup>nd</sup> stage estimates.

Table 3: Summary of Selected Parameters from Posterior Distributions: 2<sup>nd</sup> Stage IV Decomposition<sup>a</sup>

Parameter	Posterior Mean	Posterior Std. Deviation	Mean/Std. Deviation	Posterior Median
$\gamma_0$	-4.081	0.2225	-18.34	-4.072
$\gamma_{\text{secchi}}$	0.1122	0.0867	1.294	0.1128
$\gamma_{\text{chlorophyll}}$	-0.0064	0.0019	-3.36	-0.0064
$\gamma_{\text{congestion}}$	-55.91	4.29	-13.00	-55.78

<sup>a</sup>Calculated via IV regressions of each first stage draw of the intercepts on the site characteristics. Summaries are calculated from the resulting 800 sets of 2<sup>nd</sup> stage estimates.

Table 4: Point Estimates for Welfare Effects<sup>a</sup>

<u>Counterfactual Scenario</u>	<u>Partial Equilibrium Estimate</u>		<u>General Equilibrium Estimate</u>	
	Sample mean	Sample Median	Sample Mean	Sample Median
<u>Scenario 1</u> : Loss of nine highly popular sites.	-\$630.40	-\$67.64	-\$726.11	-\$142.84
<u>Scenario 2</u> : Loss of nine moderately popular sites	-\$51.86	\$0.00	-\$58.21	-\$5.08
<u>Scenario 3</u> : Widespread small quality improvements	\$207.92	\$146.01	\$87.28	\$61.16
<u>Scenario 4</u> : Localized large quality improvements	\$50.23	\$17.14	\$21.09	\$7.57

<sup>a</sup>Measured in dollars per active lake visitor person per year

Table 5: Posterior Summary for Intercepts and Variance Parameters

Site	Posterior Mean $\alpha_j$	Posterior std. Deviation $\alpha_j$	Posterior Mean $\sigma_j^2$	Posterior Std. Deviation $\sigma_j^2$
site1	-6.853	0.5698	21.6205	2.8972
site2	-3.9804	0.5177	16.5106	2.4113
site3	-3.5416	0.472	6.5615	1.2099
site4	-4.5809	0.5179	7.9619	1.3855
site5	-7.1232	0.6516	25.9651	3.5893
site6	-7.4964	0.7112	49.7431	5.6995
site7	-4.6257	0.4944	8.6021	1.4533
site8	-7.5566	0.7139	43.9953	5.5753
site9	-7.0336	0.721	98.8242	10.8453
site10	-4.0317	0.7095	64.0104	7.6369
site11	-7.2933	0.7324	49.9359	6.0285
site12	-3.9824	0.5992	19.1432	2.7964
site13	-3.9182	0.5773	11.292	1.9846
site14	-6.643	0.6569	26.8866	3.6205
site15	-3.5827	0.5861	21.062	3.005
site16	-7.0708	0.6815	56.1644	6.6652
site17	-4.594	0.6589	26.8254	3.8257
site18	-7.2464	0.6727	26.7819	3.6761
site19	-3.5336	0.5347	16.9989	2.4502
site20	-5.8983	0.5825	17.0884	2.6514
site21	-6.4162	0.7842	124.3539	13.5503
site22	-4.1526	0.6058	18.1337	2.8325
site23	-5.2395	0.7264	116.3501	12.4506
site24	-3.4916	0.4825	10.2819	1.6842
site25	-5.0832	0.6614	22.852	3.4256
site26	-6.3228	0.5336	19.2275	2.6721
site27	-4.5309	0.6175	31.509	4.0606
site28	-7.1968	0.6494	27.5623	3.7272
site29	-3.6768	0.5184	9.8924	1.8605
site30	-6.8934	0.6903	32.5796	4.5013
site31	-6.1151	0.6361	21.9603	3.3361
site32	-3.5645	0.6951	74.1869	8.4062
site33	-7.0511	0.7023	37.253	5.6784
site34	-3.143	0.3565	3.0317	0.6714
site35	-5.7108	0.667	33.1604	4.24
site36	-2.5374	0.369	4.0039	0.7492
site37	-7.7321	0.6936	52.9631	6.14
site38	-4.6876	0.4236	5.2954	0.9961
site39	-6.2413	0.6191	15.0001	2.5055
site40	-5.7511	0.6308	27.362	3.7193
site41	-5.5443	0.5887	16.054	2.2652
site42	-6.9314	0.6694	21.339	3.3877
site43	-7.01	0.6769	26.6037	3.9212
site44	-8.4337	0.7326	41.1814	5.4627
site45	-3.8146	0.3784	3.303	0.7044
site46	-5.4035	0.6383	23.6429	3.6075
site47	-3.9198	0.5969	18.939	2.8882
site48	-7.5834	0.6577	37.7102	4.9194
site49	-7.3969	0.7128	39.0452	5.3381
site50	-6.0868	0.6992	28.6422	4.2218
site51	-6.0373	0.6134	25.8125	3.5502
site52	-6.5316	0.6692	35.1292	4.7759
site53	-5.8525	0.71	40.6978	5.4499
site54	-4.6094	0.6228	15.7547	2.4939
site55	-5.317	0.6624	28.9138	4.0891
site56	-7.4607	0.6787	26.8651	3.8537
site57	-6.7462	0.6406	25.4104	3.5819

Table 5: Posterior Summary for Intercepts and Variance Parameters

Site	Posterior Mean $\alpha_j$	Posterior std. Deviation $\alpha_j$	Posterior Mean $\sigma_j^2$	Posterior Std. Deviation $\sigma_j^2$
site58	-6.8167	0.8778	69.6435	8.7107
site59	-6.6254	0.7072	92.0742	10.3736
site60	-7.13	0.6678	31.5164	4.32
site61	-4.0556	0.6159	18.9596	2.9636
site62	-7.1759	0.7303	38.1587	5.3751
site63	-5.6861	0.8047	37.9029	5.6098
site64	-3.1248	0.476	4.2605	0.981
site65	-2.4601	0.5749	14.8261	2.3857
site66	-4.2242	0.5623	11.2038	2.0147
site67	-6.3264	0.7774	41.5089	5.939
site68	-6.4949	0.8175	38.582	5.652
site69	-5.6505	0.7156	21.5728	3.3974
site70	-3.728	0.5263	11.1625	1.9554
site71	-4.483	0.7614	44.4682	6.0189
site72	-7.1417	0.687	29.4436	4.2063
site73	-4.5442	0.551	10.333	1.7792
site74	-5.2364	0.678	32.8502	4.4143
site75	-5.0529	0.7638	33.7347	5.1076
site76	-8.4861	0.8132	44.8233	6.2416
site77	-3.2839	0.4585	4.7608	1.0382
site78	-7.1689	0.6732	20.3329	3.2558
site79	-2.5307	0.4015	1.7805	0.4951
site80	-6.0551	0.553	12.0678	1.9475
site81	-3.7554	0.5977	11.5057	2.2217
site82	-5.5906	0.5478	9.5444	1.7068
site83	-4.1143	0.5724	9.1745	1.7158
site84	-5.1554	0.6426	14.0223	2.3759
site85	-3.8837	0.5858	10.5073	2.009
site86	-2.7552	0.535	4.9138	1.1047
site87	-6.5936	0.7421	35.0382	4.8049
site88	-3.7051	0.5349	8.819	1.6471
site89	-7.3492	0.7675	25.5976	4.1353
site90	-7.8308	0.7873	44.3108	6.075
site91	-3.7418	0.6025	12.2716	2.0722
site92	-7.8576	0.8142	63.3195	8.7261
site93	-6.728	0.8084	30.2937	4.7713
site94	-6.9139	0.7567	36.3674	5.1407
site95	-8.0552	0.8184	110.3577	11.8634
site96	-7.7719	0.7616	33.6603	4.7278
site97	-7.8101	0.8437	115.6849	13.7802
site98	-7.7368	0.7068	27.2369	4.0248
site99	-8.8789	0.8622	50.4859	7.1706
site100	-6.029	0.5733	8.7578	1.6529
site101	-5.7964	0.7613	135.7294	15.2796
site102	-5.7679	0.664	13.4248	2.5549
site103	-3.9927	0.7187	23.3549	3.8011
site104	-4.9423	0.6581	18.629	3.0432
site105	-5.0798	0.7101	17.3794	3.1948
site106	-6.2556	0.6396	13.7435	2.4165
site107	-6.3561	0.6834	17.4409	2.8836
site108	-6.0069	0.8007	18.8009	3.4047
site109	-7.1128	0.8412	65.8239	8.657
site110	-7.3213	0.7809	35.4324	5.141
site111	-4.6368	0.5793	8.9574	1.75
site112	-7.9943	0.7794	51.582	6.5441
site113	-4.1814	0.6463	10.4926	2.0675
site114	-2.5684	0.467	5.0929	1.0555
site115	-7.9598	0.8528	51.4775	6.9238

Table 5: Posterior Summary for Intercepts and Variance Parameters

Site	Posterior Mean $\alpha_j$	Posterior std. Deviation $\alpha_j$	Posterior Mean $\sigma_j^2$	Posterior Std. Deviation $\sigma_j^2$
site116	-8.6359	0.9268	37.511	6.1373
site117	-5.28	0.7707	29.4527	4.494
site118	-9.9038	0.9543	64.0853	8.7494
site119	-5.6115	0.7516	30.5258	4.6671
site120	-7.2441	0.8385	47.4747	6.4673
site121	-4.7558	0.9157	98.7864	12.4806
site122	-7.4532	0.7654	25.5415	3.7944
site123	-5.3224	0.5757	9.008	1.5731
site124	-3.5836	0.541	3.6977	0.9531
site125	-4.1893	0.6111	11.6769	2.0662
site126	-3.9118	0.6452	8.4505	1.7459
site127	-4.6318	0.7037	13.6563	2.4983
site128	-5.0304	0.6462	18.0508	2.8207

Table 6: PE and GE welfare estimates by policy scenario and intensity of congestion effect, Totals specification

Scenario	Half Congestion Effect			Full Congestion Effect			Double Congestion Effect		
	<u>PE</u>	<u>GE</u>	<u>%Diff</u>	<u>PE</u>	<u>GE</u>	<u>%Diff</u>	<u>PE</u>	<u>GE</u>	<u>%Diff</u>
1	-552.3	-549.1	-0.6	-524.5	-527.2	0.5	-469.0	-506.7	7.4
2	-195.4	-193.3	-1.1	-193.3	-189.9	-1.8	-189.0	-184.2	-2.6
3	157.7	126.2	-25.0	153.6	106.7	-44.0	145.3	88.1	-64.9
4	59.7	49.4	-20.9	58.2	41.8	-39.2	55.2	32.8	-68.1

Table 7: PE and GE welfare estimates by policy scenario and Totals vs. Shares Congestion Specification, Full Congestion Effect

Scenario	Totals Specification			Shares Specification		
	<u>PE</u>	<u>GE</u>	<u>%Diff</u>	<u>PE</u>	<u>GE</u>	<u>%Diff</u>
1	-524.5	-527.2	0.5	-580.0	-618.3	6.2
2	-193.3	-189.9	-1.8	-197.6	-212.1	6.9
3	153.6	106.7	-44.0	161.9	172.2	6.0
4	58.2	41.8	-39.2	61.2	56.0	-9.2

Figure 1: Histograms Showing Empirical Distributions

