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Incorporating demand shifters in the Almost Ideal demand system

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Abstract

Intercepts of share equations generally include demand shift variables. In the Almost Ideal demand system and related models, this results in estimates that depend on units of measurement. Solutions to this problem are identified and discussed.

Keywords: Almost Ideal demand system; Generalized Almost Ideal demand system; Demand shifters; Scaling

JEL classification: D12; Q11

1. Introduction

In consumer demand analysis, variables other than prices and income are often included in demand equations. For instance, in studies using cross-sectional data, demographic variables might be included, and in studies using time series data, trends and seasonal variables might be included. Researchers must be careful to incorporate these demand shift variables in a manner that maintains the theoretical properties of the model. Importantly, the estimated demand system must be 'Closed Under Unit Scaling' (CUUS), a property that ensures that estimated economic effects are invariant to the scaling of the data (Pollak and Wales, 1992, p. 70). In applications of the very popular Almost Ideal demand system, the intercepts of the expenditure share equations are commonly expressed as linear functions of other explanatory variables, as originally suggested by Deaton and Muellbauer (1980, p. 320). Below we show that the resulting model is no longer consistent with CUUS. We explore two

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specific alternative approaches for incorporating demand shifters and preserving CUUS while retaining the other desirable features of the Almost Ideal model.

2. The problem

The Almost Ideal model is derived from the expenditure function

$$E(p,u) = \exp[a(p) + ub(p)], \tag{1}$$

where

$$a(p) = \ln P = \delta + \alpha'\hat{p} + (1/2)\hat{p}'\Gamma\hat{p} \tag{2}$$

and

$$b(p) = \prod_{j=1}^{n} p_j^{\beta_j}. \tag{3}$$

The corresponding vector of share equations is

$$w = \alpha + \Gamma\hat{p} + \beta \ln(M/P), \tag{4}$$

where w denotes the vector of budget shares, p denotes the vector of prices, M is total expenditure, $\hat{p}$ denotes the vector of logarithms of p, and the parameters to be estimated are δ, the vectors α and β, and the matrix Γ. Let ι denote a unit vector. To satisfy homogeneity, adding-up, and symmetry, the following restrictions must hold:

$$\iota'\alpha = 1, \quad \iota'\Gamma = 0, \quad \iota'\beta = 0, \quad \Gamma' = \Gamma.$$

When the intercepts (α) of the share equations are expressed as linear functions of some demand shift variable Z, α is redefined as $\alpha = \alpha_0 + \lambda \cdot Z$, and the restrictions $\iota'\alpha_0 = 1$ and $\iota'\lambda = 0$ are required for adding-up. The effects of changes in Z are thus seen as changes in the parameters of the expenditure function, and integrability is preserved in the share equations by letting each element of α vary with Z, while retaining the adding-up restriction. The share equations become

$$w = \alpha_0 + \lambda \cdot Z + \Gamma\hat{p} + \beta \ln M - \beta[\delta + (\alpha_0 + \lambda \cdot Z)'\hat{p} + (1/2)\hat{p}\Gamma\hat{p}]. \tag{5}$$

Unfortunately, augmenting α in this fashion results in estimated economic effects that are not invariant to units of measurement of the prices and quantities of the goods in the model, a shortcoming that has not been documented previously. Specifically, a disproportionate change in the units of measurement for any or all of the goods changes the estimates of Γ and β so that elasticities and consumer welfare measures also change. Consider changing the units of measurement for quantities, using $\tilde{q}_i = q_i\pi_i$, and $\tilde{p}_i = p_i/\pi_i$ ($\pi_i > 0$). In the model without Z, when we replace $\hat{p}$ with $\hat{p} - \hat{\pi}$, where $\hat{\pi}_i = \ln \pi_i$, the share equations (4) become

$$
\begin{aligned}
w &= \tilde{\alpha} + \tilde{\Gamma}(\hat{p} - \hat{\pi}) + \tilde{\beta} \ln(M/\tilde{P}) \\
&= \tilde{\alpha} + \tilde{\Gamma}(\hat{p} - \hat{\pi}) + \tilde{\beta} \ln M - \tilde{\beta}[\tilde{\delta} + \tilde{\alpha}'(\hat{p} - \hat{\pi}) + (1/2)(\hat{p} - \hat{\pi})'\tilde{\Gamma}(\hat{p} - \hat{\pi})] \\
&= \tilde{\alpha} - \tilde{\Gamma}\hat{\pi} + \tilde{\Gamma}\hat{p} + \tilde{\beta} \ln M - \tilde{\beta}[\tilde{\delta} - \tilde{\alpha}'\hat{\pi} + (1/2)\hat{\pi}'\tilde{\Gamma}\hat{\pi} + \tilde{\alpha}'\hat{p} + (1/2)\hat{p}'\tilde{\Gamma}\hat{p} - \hat{\pi}'\tilde{\Gamma}\hat{p}] \\
&= \tilde{\alpha} - \tilde{\Gamma}\hat{\pi} + \tilde{\Gamma}\hat{p} + \tilde{\beta} \ln M - \tilde{\beta}[\tilde{\delta} - \tilde{\alpha}'\hat{\pi} + (1/2)\hat{\pi}'\tilde{\Gamma}\hat{\pi} + (\tilde{\alpha} - \tilde{\Gamma}\hat{\pi})'\hat{p} + (1/2)\hat{p}'\tilde{\Gamma}\hat{p}] \\
&= \alpha + \Gamma\hat{p} + \beta \ln M - \beta[\delta + \alpha'\hat{p} + (1/2)\hat{p}'\Gamma\hat{p}],
\end{aligned} \tag{6}
$$

provided that $\alpha = \tilde{\alpha} - \tilde{\Gamma}\hat{\pi}$ and $\delta = \tilde{\delta} - \tilde{\alpha}'\hat{\pi} + (1/2)\hat{\pi}'\Gamma\hat{\pi}$. Hence, rescaling leaves $\tilde{\Gamma} = \Gamma$ and $\tilde{\beta} = \beta$, and α and δ adjust to changes in units of measurement, a familiar result when constants are added to regressors. Predicted shares, elasticities, and welfare measures are thus invariant to such scaling choices.

The same invariance is not achieved when a similar rescaling is done in the augmented demand system in (5), which becomes

$$
\begin{aligned}
w = {} & \tilde{\alpha}_0 + \tilde{\lambda} \cdot Z + \tilde{\Gamma}(\hat{p} - \hat{\pi}) + \tilde{\beta} \ln M \\
& - \tilde{\beta}[\tilde{\delta} + \tilde{\alpha}_0'(\hat{p} - \hat{\pi}) + Z \cdot \tilde{\lambda}'(\hat{p} - \hat{\pi}) + (1/2)(\hat{p} - \hat{\pi})'\tilde{\Gamma}(\hat{p} - \hat{\pi})].
\end{aligned} \tag{7}
$$

Collecting the terms that involve $\hat{\pi}$, as in (6), the share equations become

$$
w = \overline{\alpha}_0 + \tilde{\lambda} \cdot Z + \tilde{\Gamma}\hat{p} + \tilde{\beta} \ln M - \tilde{\beta}[\overline{\delta} + (\overline{\alpha}_0 + \tilde{\lambda} \cdot Z)'\hat{p} + (1/2)\hat{p}'\tilde{\Gamma}\hat{p}], \tag{8}
$$

where $\overline{\alpha}_0 = \tilde{\alpha}_0 - \tilde{\Gamma}\hat{\pi}$ and $\overline{\delta} = \tilde{\delta} - \tilde{\alpha}_0'\hat{\pi} + (1/2)\hat{\pi}'\tilde{\Gamma}\hat{\pi} - Z\tilde{\lambda}'\hat{\pi}$. Notice that $\overline{\delta}$ is not constant, because the term $Z\tilde{\lambda}'\hat{\pi}$ varies across the data, and so this model with the prices rescaled is not equivalent to the original model. As a result, the other parameters (i.e., Γ, β, and λ) are affected by the scaling, and hence the price and income elasticities and consumer welfare measures now depend on the units chosen for measuring quantities.[2] This means that any empirical results from an Almost Ideal model augmented in this way are conditioned on some choice of quantity units.

3. Possible solutions

Closure Under Unit Scaling imposes the condition that if some parameters depend on scaling, then others may need to as well. Specifically, when we make α a linear function of demand shifters, CUUS requires that we also make δ a linear function of the same demand shifters. That is, if one adopts $\alpha = \alpha_0 + \lambda \cdot Z$, then one must also adopt $\delta = \delta_0 + \nu \cdot Z$.[3] The augmented share equations then become

$$
w = \alpha_0 + \lambda \cdot Z + \Gamma\hat{p} + \beta \ln M - \beta[\delta_0 + \nu \cdot Z + (\alpha_0 + \lambda \cdot Z)'\hat{p} + (1/2)\hat{p}\Gamma\hat{p}]. \tag{9}
$$

After rescaling the quantity units, as above, the share equation is

[2] The two versions of the model are equivalent only if the scale factor π_i does not vary across the goods, so that $\pi_i = \pi \ \forall i$, which implies that $\lambda'\hat{\pi} = 0$ (since $\iota'\lambda = 0$ for adding-up).

[3] We are grateful to an anonymous referee for pointing out this alternative.

$$w = \tilde{\alpha}_0 + \tilde{\lambda} \cdot Z + \tilde{\Gamma}(\hat{p} - \hat{\pi}) + \tilde{\beta} \ln M - \tilde{\beta}[\tilde{\delta}_0 + \tilde{\nu} \cdot Z + \tilde{\alpha}_0'(\hat{p} - \hat{\pi}) + Z \cdot \tilde{\lambda}'(\hat{p} - \hat{\pi}) + (1/2)(\hat{p} - \hat{\pi})'\tilde{\Gamma}(\hat{p} - \hat{\pi})]. \tag{10}$$

Collecting the terms that involve $\hat{\pi}$ results in

$$w = \alpha_0 + \lambda \cdot Z + \Gamma\hat{p} + \beta \ln M - \beta[\delta_0 + \nu \cdot Z + (\alpha_0 + \lambda \cdot Z)'\hat{p} + (1/2)\hat{p}'\tilde{\Gamma}\hat{p}], \tag{11}$$

where $\alpha_0 = \tilde{\alpha}_0 - \tilde{\Gamma}\hat{\pi}$, $\delta = \tilde{\delta} - \alpha_0'\hat{\pi} + (1/2)\hat{\pi}'\tilde{\Gamma}\hat{\pi}$, and $\nu = \tilde{\nu} - \tilde{\lambda}'\hat{\pi}$. Notice that α_0, δ, and ν are all constant and are merely rescaled, and Γ, β, and λ are unchanged (i.e., $\tilde{\Gamma} = \Gamma$, $\tilde{\beta} = \beta$, and $\tilde{\lambda} = \lambda$). The desired invariance property is achieved.

Deaton and Muellbauer (1980) pointed out that it may be difficult to estimate δ, and subsequent experience has borne out that prediction. Such difficulties would be compounded if we were to make δ itself a function of other variables. Hence, in many instances, it will not be feasible to estimate a model in which demand shifters augment both α and δ, and an alternative solution must be found.

General methods for incorporating demand shifters and preserving CUUS in demand systems include scaling, translating, and other modifying functions, as described by Lewbel (1985). Among these theoretical alternatives, we would like to identify a practicable alternative that will preserve the other desirable features of the Almost Ideal model, and will also allow demand shifters to be incorporated parsimoniously, and flexibly. One such alternative is to adopt a generalized version of the Almost Ideal model, the Generalized Almost Ideal (GAI) model, first derived by Bollino (1987). The expenditure function used to characterize the GAI model allows for a portion of total expenditures to be allocated to pre-committed quantities, which are unobservable and must be estimated along with the other parameters in the demand system. The GAI expenditure function can be expressed as

$$E(p,u) = p'c + E^*(p,u), \tag{12}$$

where c denotes the vector of pre-committed quantities and $E^*(p,u)$ is the expenditure function for supernumerary (beyond pre-committed) expenditures, defined to be of the Almost Ideal form, as in Eqs. (1)–(3). In the GAI model, the share equation for the budget share of good i is

$$w_i = \frac{p_i c_i}{M} + \frac{M^*}{M}\left(\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln(M^*/P)\right), \tag{13}$$

where $M^* = M - p'c$ and $\ln P$ is defined as in (2).

The GAI model can, of course, be viewed as a generalization of the Linear Expenditure System (LES), in which the marginal budget shares are no longer constant, but are instead of the Almost Ideal form. The pre-committed quantities are independent of prices and expenditure, suggesting that augmenting the c vector by making each pre-committed quantity a function of Z could avoid the scaling problems described earlier. Such an approach was referred to as ‘demographic translating’ by Pollak and Wales (1992, p. 12), who showed how to use it to incorporate demographic variables in the LES.

The same approach can be applied in the GAI model to incorporate any type of demand shifter, producing estimates that are invariant to changes in quantity units. To see this, replace each pre-committed quantity c_i with a linear function of Z:

$$c_i = c_{i0} + \lambda_i Z. \tag{14}$$

Then the share equations will take the form

$$w_i = \frac{p_i[c_{i0} + \lambda_i Z]}{M} + \frac{M^*}{M}\left(\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln(M^*/P)\right). \tag{15}$$

When the quantity units are redefined so that prices are rescaled, once again by dividing by π_i, the demand system becomes

$$w_i = \frac{(p_i/\pi_i)[\tilde{c}_{i0} + \tilde{\lambda}_i Z]}{M} + \frac{M^*}{M}\left(\tilde{\alpha}_i + \sum_{j=1}^{n} \tilde{\gamma}_{ij} \ln(p_j/\pi_j) + \tilde{\beta}_i \ln(M^*/P)\right). \tag{16}$$

The marginal budget share in parentheses is the right-hand side of the share equation of the Almost Ideal demand system; we showed above that this term is invariant to changes in quantity units. In addition, M is fixed and M^* changes only if the pre-committed expenditures, $p'c$, change. To see that pre-committed expenditures do not change when quantity units change, consider the first term:

$$\frac{p_i[c_{i0} + \lambda_i Z]}{M} = \frac{(p_i/\pi_i)[\pi_i \cdot (c_{i0} + \lambda_i Z)]}{M} = \frac{(p_i/\pi_i)[\tilde{c}_{i0} + \tilde{\lambda}_i Z]}{M}. \tag{17}$$

Hence, the new $\tilde{c}_{i0}$ is simply $c_{i0}\pi_i$ and the new $\tilde{\lambda}_i$ is simply $\pi_i \lambda_i$. Thus, changing the units of measurement in the GAI demand model is of no consequence. Augmenting the pre-committed quantities in this fashion does not appear to imply any restrictions on how Z affects the demand for any particular good.

4. Conclusion

In one of the most popular models used in demand analysis, the Almost Ideal demand system, it is common to make the intercepts of the share equations linear functions of demand shift variables, a procedure that means results are not invariant to disproportionate changes in units of measurement. We have described two specific alternative approaches for incorporating demand shifters in the Almost Ideal model that overcome this invariance problem.

The more appealing approach, from an empirical standpoint, involves adopting the Generalized Almost Ideal model, and allowing the pre-committed quantities to be linear functions of demand shift variables. This approach allows the demand shifters to be included in a fashion that is flexible, parsimonious, and maintains the model's invariance to changes in units of measurement. It is easily seen that similar results will be found with related models of the PIGLOG class. For instance, Lewbel's (1989) model that nests the Almost Ideal and Translog models will exhibit the same dependency on units as the Almost Ideal model. One solution is to include demand shift variables as modifications of the pre-committed quantities, in Bollino and Violi's (1990) generalization of Lewbel's model.

References

Bollino, C.A., 1987. GAIDS: a generalized version of the Almost Ideal demand system. Economics Letters 23, 199–203.
Bollino, C.A., Violi, R., 1990. GAITL: a generalized version of the Almost Ideal and translog demand systems. Economics Letters 34, 127–129.
Deaton, A.S., Muellbauer, J., 1980. An Almost Ideal demand system. American Economic Review 70, 312–326.
Lewbel, A., 1989. Nesting the AIDS and translog demand systems. International Economic Review 30, 349–356.
Lewbel, A., 1985. A unified approach to incorporating demographic or other effects into demand systems. Review of Economic Studies, 1–18.
Pollak, R.A., Wales, T.J., 1992. Demand System Specification and Estimation. Oxford University Press.